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RESEARCH MEMORANDUM

A SIMPLIFIED METHOD FOR EVALUATING JET-PROPULSION-SYSTEM COMPONENTS IN TERMS OF AIRPLANE PERFORMANCE

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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A SIMPLIFIED METHOD FOR EVALUATING JET-PROPULSION-SYSTEM COMPONENTS IN TERMS OF AIRPLANE PERFORMANCE

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SUMMARY

It is often impossible to determine, from an engine study alone, which of two aircraft engines is superior for a given flight application. Consideration of weight, drag, thrust, and efficiency individually may give contradictory results. This difficulty is commonly experienced in research on propulsion-system components, particularly the inlet diffuser and the exhaust nozzle.

A method is developed to provide a simple means of comparing engine components on the basis of either range or the margin of thrust over drag. The equations developed include not only the variation in thrust coefficient and specific impulse but also the change in engine weight and drag. Four general cases are considered:

(1) Fixed-size airplane with constant gross weight

(2) Fixed-size airplane with variable gross weight

(3) Variable-size airplane with constant payload weight

(4) Variable-size airplane with constant ratio of payload to gross weight

Calculated performance data of representative turbojet and ram-jet engines are presented so that the investigator interested in the effects of diffuser pressure recovery or nozzle velocity coefficient can apply the method without first carrying out an engine cycle analysis.

Several numerical examples are worked out to illustrate use of the method and typical applications.

INTRODUCTION

The performance of an aircraft - for example, its range - is affected by the fuel economy, thrust, weight, and drag of the installed engine. Frequently, a physical change in some propulsion-system component will improve one of these engine parameters such as thrust but also worsen another such as weight. The researcher or designer in the field of propulsion systems is then unable to tell, on the basis of the engine parameters alone, whether or not the net effect of the component modification would be beneficial to airplane performance. The solution to this problem may be found through a performance analysis of the airplane for the mission desired. Such analysis may, however, be beyond the purview or inclination of the researcher. A simpler, more general approach is desirable for a rapid, although less precise, evaluation.

This report presents a simple method of indicating the net effect of a simultaneous change in two or more engine performance indices (i.e., thrust, specific impulse, weight, or drag) on airplane performance. Two measures of airplane performance were selected: (1) range, because of its importance to most airplane missions, and (2) maximum engine-thrust minus airplane drag, because of its importance to such characteristics as rate of climb and maneuverability.

Particular emphasis is placed on applying the method to inlets and exhaust nozzles in the derivation and discussion. To facilitate use of the method with ram jets and turbojets, representative performance data of the required form are presented. Numerical examples are also given to illustrate typical applications of the method.

References 1 and 2 present methods for calculating engine performance and performance derivatives that may be adapted for use in the equations of the present report. Reference 3 came to the attention of the authors after the present work was completed. It develops relations between diffuser pressure recovery and drag and airplane range. The development of the present report is considerably broader than and somewhat different in point of view from reference 3.

ANALYSIS

The analysis is divided into two parts. The first deals with the airplane range, and the second with airplane acceleration potential. (The airplane acceleration potential is defined as the maximum engine thrust minus airplane drag in steady level flight divided by the airplane gross weight.) In both sections the effect of several assumptions regarding the airplane size and weight is considered.

Throughout this report the term "engine" is defined to include the inlet diffuser, exhaust nozzle, and nacelle (if externally mounted).

Range Evaluation

The method developed to evaluate engine performance in terms of airplane range is basically a sensitivity analysis of the Breguet range equation, which considers the engine and airframe as an integrated combination. For jet aircraft the Breguet equation may be written

$$\mathcal{R} = V\bar{I}\,\frac{L}{D_t}\,\ln\left[\frac{1-\left(\frac{W_f}{W_g}\right)_\alpha}{1-\frac{W_f}{W_g}}\right] \tag{1}$$

The symbols used in this report are defined in appendix A.

In the subsequent development, the flight speed V is assumed to be constant. This corresponds to constant flight Mach number in the stratosphere. The fuel required to accelerate and climb to cruise conditions $(W_f/W_g)_\alpha$ is also assumed to be a constant - that is, independent of changes that primarily affect cruise conditions. Logarithmic differentiation with respect to any engine parameter X (e.g., X might be the inlet pressure recovery) yields

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{\bar{I}\,\frac{L}{D_t}}\,\frac{d\left(\bar{I}\,\frac{L}{D_t}\right)}{dX} + \frac{1}{k}\,\frac{d\left(\frac{W_f}{W_g}\right)}{dX} \tag{2a}$$

where

$$k = \left(1-\frac{W_f}{W_g}\right)\ln\left[\frac{1-\left(\frac{W_f}{W_g}\right)_\alpha}{1-\frac{W_f}{W_g}}\right]$$

Equation (2a) permits finding the relative change in range $d\mathcal{R}/\mathcal{R}$ resulting from a small change dX in some engine parameter (provided, of course, that all the terms can be evaluated). Alternatively, it may often be useful to find the change in either engine weight or drag which, in conjunction with the variation dX, results in the same airplane range. This value, termed the break-even condition, may be found by taking $d\mathcal{R}$ equal to zero. The condition $\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = 0$ is also the condition for determining the value of X that will yield maximum range.

To be readily used by the propulsion-systems engineer, equation (2a) must be expressed in terms of the usual indices of engine performance. In this report, thrust coefficient, specific impulse, weight, and drag coefficient are used.

The development in appendix B shows that equation (2a) may be written

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{\bar{I}}\frac{d\bar{I}}{dX} + \frac{1}{k}\frac{d}{dX}\left(\frac{W_f}{W_g}\right) \tag{2b}$$

which eliminates the airplane lift-drag ratio and involves only an engine performance term and a term involving the fuel weight. Evaluation of these two terms depends on the airplane being considered. The required mathematical development is detailed in appendix B; for simplicity, the results are summarized in table I and discussed in the following sections.

Fixed-size, fixed-gross-weight airplane. - This section considers the case of an airplane with both airframe and engine of fixed size and with initial gross weight held constant even when the engine weight is varied. Appendix B shows that the term $d\bar{I}/dX$ in equation (2b) is to be evaluated at a constant value of thrust coefficient for this case. Also, differentiating the summation of airplane weights (with the assumption of a fixed payload weight)

$$W_g = W_{st} + W_{pl} + W_e + W_f \tag{3}$$

gives

$$\frac{dW_e}{dX} = -\frac{dW_f}{dX} \tag{4}$$

This requires that the airplane take off with partially empty fuel tanks when the modification dX increases the engine weight. Conversely, when the engine weight is reduced, the weight saved is assumed to be put into additional fuel (but with negligible change in structural weight or drag). The general result for this case is given as equation (B18) in table I(a). Equations (T2) to (T5) are given to illustrate how the general equation might be rewritten for several particular applications. In equations (T2) and (T3), for example, the engine parameter X is taken as the inlet pressure recovery $\mathcal{P}$. Equation (T2) permits calculating the change in range resulting from a variation in pressure recovery that has affected the engine thrust, specific impulse, and weight, but not the drag. Equation (T3) gives the more common case where the variation in pressure recovery affects the diffuser drag but not the weight. Equations (T4) and (T5) give similar relations for the effect on range of changes in nozzle velocity coefficient. Equations (T2) and (T3) follow directly from equation (B18) by setting dC_D and dW_e equal to zero, respectively. Equations (T4) and (T5) are obtained in the same manner, recognizing the fact that changes in the nozzle affect C_F and I in the same proportion (see the derivation of eq. (B22) in appendix B). The corresponding break-even relation is obtained from any of these given equations by merely setting $d\mathcal{R}$ equal to zero.

Fixed-size, variable-gross-weight airplane. - This section considers the case of a fixed-size airplane in which the fuel weight is not changed even when an engine modification dX results in a change in engine weight. Differentiation of the airplane weight equation in the preceding section now gives

$$\frac{dW_g}{dX} = \frac{dW_e}{dX} \tag{5}$$

for $dW_f/dX = 0$. The takeoff gross weight of the airplane thus varies as the engine weight is varied. The resulting general range equation (B24) is presented in table I(b). As before, simplified equations can be obtained from the general equation for particular engine components.

Variable-size and gross-weight airplane, constant payload and acceleration potential. - When the engine is modified in a fixed-size airplane, not only the range but also the acceleration potential (discussed in the next section) is changed. It is assumed in this section that the acceleration potential of the airplane is held constant when the engine is modified. In order to do this, the geometric proportions of the airframe are assumed unchanged, but its size and weight are allowed to vary somewhat as the engine is modified. This variation is applicable to airplanes still in the preliminary design stage. Engine size, however, is considered fixed. The resulting variations in $C_{\mathcal{F}}$ and W_f/W_g for this case are given in equations (B34) and (B40) and are not repeated here because of their complexity. Equation (T6) of table I(c) is the general result for the case where the payload weight in pounds is assumed constant.

Variable-size and gross-weight airplane, constant percent payload and constant acceleration potential. - This case is the same as that discussed in the previous section, except that the payload is assumed to remain a constant percentage of the airplane gross weight rather than a constant that is independent of gross weight. The general result for this case is given in equation (T8) of table I(c). Again, simplified equations can be written for application to particular engine components.

Evaluation of Acceleration Potential

Acceleration potential G is defined in this report as maximum engine propulsive thrust minus airplane steady-level-flight drag divided by airplane gross weight:

$$G = \frac{\mathcal{F}_{max} - D_t}{W_g} \tag{6}$$

where G may be evaluated for the initial gross weight or for any subsequent value of W_g during flight. This acceleration factor is an indication of the airplane's climb, acceleration, and combat capabilities. It thus is probably of greatest interest for interceptor applications. 4043

Engine modifications usually affect not only the cruising performance but also the maximum thrust of the engine. Therefore, both the airplane range and acceleration potential are changed. The effect of engine modifications on acceleration potential for the case where the airplane size is fixed is considered in appendix B. The general expressions for this case are given in table II.

By varying airplane size it is possible to vary the value of acceleration potential or to hold a constant value while modifying the engine. (As already discussed, this latter condition was specified when deriving the range equations of table I(c).)

APPLICATION OF METHOD

Engine and Airplane Parameters

In order to evaluate numerically the equations that have been derived, it is necessary to know the values of the engine thrust coefficient and specific impulse and their derivatives with respect to the engine parameter X that is of interest. Several constants describing the airplane must also be known. Some representative values of these engine and airplane variables are presented in this section to facilitate use of the equations. Engine derivatives are given for the cases where the engine parameter X to be studied is either the inlet pressure recovery or the exhaust-nozzle velocity coefficient. Studies of other engine parameters, such as compressor efficiency, would first require determination of similar derivatives by means of a cycle analysis or from experimental data.

<u>Engine performance</u>. - A limited amount of data is presented herein for a turbojet engine and a ram-jet engine operating over a range of supersonic flight speeds in the stratosphere. The performance for both

engines is based on the component assumptions given in appendix C. The curves represent either a series of engines each operating at its design point or a single engine with a continuously variable inlet and nozzle.

Figure 1(a) shows the internal thrust coefficient C_F and specific impulse I of the turbojet engine for a range of afterburner temperatures up to 3500° R and for flight Mach numbers from 1.0 to 3.0. Figure 1(b)
4043 gives the derivative of the specific impulse with respect to the internal thrust coefficient. Figures 1(c) to (f) present the derivatives of the turbojet thrust coefficient and specific impulse with respect to the inlet pressure recovery and exhaust-nozzle velocity coefficient. Figure 2 presents similar data for a ram-jet engine up to Mach 5.0. The diffuser area ratios and assumed pressure recoveries as a function of Mach number are given in table III, and exhaust-nozzle pressure ratios and area ratios are listed in table IV.

The engine performance data presented are affected in varying degrees by the assumptions. Therefore, if performance data are available for a specific engine of interest, they should be used. However, the engine data presented herein will, in general, yield similar results for the following reasons. The results are independent of the value assumed for compressor air flow, since I is not a function of air flow and since C_F, which is a function of air flow, always enters into the equations as a ratio of thrust or drag coefficients. The derivatives $\partial I/\partial \mathcal{P}$ and $\partial C_F/\partial \mathcal{P}$ are, to a first order, independent of $\mathcal{P}$, because C_F and I are approximately linear with $\mathcal{P}$. The derivatives $\partial I/\partial C_V$ and $\partial C_F/\partial C_V$ are independent of the assumed C_V, because C_F and I are exactly linear with C_V. These are also the considerations that permit use of finite changes in $\mathcal{P}$ and C_V in the differential equations rather than infinitely small variations. In substantiation of the above argument, an example discussed later in the report shows that the derivative $dC_{D,A_0}/d\mathcal{P}$ is about the same whether calculated for the turbojet or ram-jet engine.

Generally, it will be desired to compare two designs of an engine component, neither of which is described exactly by the parameters assumed for that component in the present report. Comparison of the two given designs is accomplished by comparing each of them with the component assumed herein.

It is necessary to select the cruising combustion temperature in order to obtain values of C_F and I for use in the equations. In order to limit engine weight, the cruising temperature is usually chosen as somewhat higher than that for maximum I. This temperature is generally of the order of 2500° to 3000° R for supersonic cruising at Mach numbers up to 3.0.

Airplane characteristics. - When a specific design is not available to work with, and if primary interest is in the study of the engine components, sufficient accuracy for the application of the method of the present report is obtainable through the use of typical airplane characteristics. Table V presents estimated values of required airplane parameters that are representative of four classes of aircraft.

Illustrative Examples

Several numerical examples will be presented to illustrate how the equations are used, to illustrate the effect of the various assumptions that can be made with regard to the airplane, and to suggest typical applications of the method. 4043

Effect of airplane assumptions on relative importance of pressure recovery and weight. - Several different assumptions were made in deriving the expressions for range. First, the airplane size was considered to be fixed and the gross weight was either held constant or allowed to vary. Second, the airplane size was allowed to vary with the option of holding either the payload or the ratio of payload to gross weight constant. It is informative to compare the results obtained for these four cases.

Consider a turbojet-powered interceptor airplane flying at a Mach number of 2.0. The equations will be used to calculate the permissible increase in engine weight to maintain the same range if the inlet pressure recovery is improved by 0.01 and the drag does not change. (Although the calculated result may be interpreted several ways, the implication here is that the improved inlet pressure recovery is achieved by a mechanical device such as a variable-angle ramp that increases the inlet weight, the inlet being considered part of the engine.) Assume arbitrarily that the airplane has a gross weight of 20,000 pounds, cruises at an afterburner temperature of 2500° R, and has a maximum afterburner temperature of 3500° R. Figures 1(a) to (d) give the following values for the engine performance:

$$C_F = 1.73 \qquad \frac{\partial I}{\partial \mathscr{P}} = 700$$

$$I = 2215 \text{ lb/(lb/sec)} \qquad C_{F,max} = 2.52$$

$$\frac{\partial I}{\partial C_F} = -730 \qquad \frac{\partial C_{F,max}}{\partial \mathscr{P}} = 3.40$$

$$\frac{\partial C_F}{\partial \mathscr{P}} = 2.48$$

The weight ratios for the airplane are taken from table V. These values for the engine and airplane are then substituted in equations (B18), (B24), (T6), and (T8) of table I, taking both $d\mathcal{P}$ and dC_D equal to zero. (Eq. (T2) of table I(a) illustrates how eq. (B18), e.g., is simplified for this particular calculation.) The results of the calculation are as follows:

Airplane assumptions	ΔW_e, lb	ΔW_g, lb
Fixed size, constant W_g	36.3	0
Fixed size, variable W_g	121.0	121.0
Variable size, constant W_{pl}	106.0	270.0
Variable size, constant W_{pl}/W_g	79.1	270.0

The smallest acceptable increase in engine weight occurs for the fixed-size airplane with constant gross weight. In this case the fuel weight is decreased to accommodate the increased engine weight. Allowing the takeoff gross weight to rise permits substantially heavier engines.

<u>Comparison of range and acceleration criteria in determining relative importance of pressure recovery and drag</u>. - In addition to range, the maneuverability and acceleration capability of an interceptor airplane are important. For the airplane in the previous example, it is desired to compare the acceptable increases in engine drag resulting from an improvement in pressure recovery for (1) constant range, and (2) constant acceleration potential. The airplane is assumed to be of fixed size and constant gross weight. Cruise afterburner temperature is 2500° R, while the thrust minus drag expression (eq. (T9) of table II) is evaluated at 3500° R. At 3500° figure 1(c) gives

$$\frac{\partial C_{F,max}}{\partial \mathcal{P}} = 3.40$$

Substituting the appropriate values in equations (T3) and (T9) of tables I(a) and II with $d\mathcal{P}$ and dG taken as zero gives the following:

Break-even condition	$dC_D/d\mathcal{P}$
Range	1.25
Thrust minus drag	3.40

There is an appreciable difference in acceptable drag rise between the two cases. The relative importance of the two criteria depends on whether the airplane is expected to cruise long distances at supersonic speeds or whether it is required only to have high maneuverability during short bursts of combat.

Graphical comparison of several inlets. - Suppose that several diffuser configurations have been developed having the characteristics at a Mach number of 2.0 that are shown in columns (1), (2), and (3) of the following table:

	(1)	(2)	(3)	(4)
Diffuser	$\mathcal{P}$	$C_{D,l}$	A_0/A_l	C_{D,A_e}
A	0.85	0.03	1.0	0.023
B	.91	.18	1.0	.148
C	.92	.19	.8	.197

(2) Drag coefficient based on inlet capture area.

(4) Drag coefficient based on compressor frontal area.

Suppose further that the turbojet interceptor airplane discussed in the preceding examples has been built with diffuser B. It is desired to know whether the same engine using diffuser A or C might afford better airplane performance.

A graphical presentation is convenient in this problem both to present the data and to compare inlets.

First, the diffuser drag coefficients (column (2)) must be based on the same area used for the engine thrust coefficients, which for the turbojet engine is the compressor frontal area. The conversion may be made by the following equation:

$$C_D = \left(\frac{\frac{A_0}{A_e}}{\mathcal{P}}\right)\frac{\mathcal{P}}{\frac{A_0}{A_l}}C_{D,l} \tag{7}$$

where the term in parentheses is given in table III for the engine previously described, and the remaining terms on the right are characteristics of the inlet under consideration. This equation correctly sizes the inlet to the engine, with the assumption that the diffuser-exit (or compressor-inlet) Mach number does not vary with φ for a given engine. (This assumption is discussed further in appendix C.) The equation also converts the inlet drag coefficient from the reference area A_l to A_e, on which C_F is based. Carrying out this calculation for the present example results in the drag coefficients given in column (4).

A figure may be constructed of diffuser drag coefficient as a function of pressure recovery (fig. 3). The break-even lines are drawn through the values of diffuser B with the slopes of 1.25 and 3.40 that were determined in the preceding example. (The shaded side of the lines indicates the unfavorable region, i.e., reduced range or reduced thrust minus drag.) Superimposing the data for diffusers A and C shows immediately whether the range or the thrust-minus-drag margin has been improved. Figure 3 shows that diffuser C has such high drag that it is poorer than diffuser B on the basis of both criteria. Use of diffuser A, however, would improve airplane range but decrease its margin of thrust over drag.

Each diffuser in this example has been described by a single point. It is also possible to plot on the same type of figure a curve for each diffuser showing the φ and C_D corresponding to different mass-flow ratios. This would then permit determination of the best operating point with respect to either range or acceleration potential. The procedure of this section is most applicable to isolated-inlet investigations.

Graphical method for selecting best inlet size and operating point. - The procedure in this section is approximate and most useful in cases where the inlet drag is combined with other drags, as is the case with a fuselage side inlet, so that the quantity of drag associated with the inlet is not separately known. One of the frequently used plots for presenting experimental inlet performance data is shown in figure 4. The drag coefficient is usually based on the model maximum cross-sectional area A_{max}. Presented on the plot is a typical set of data for a flight Mach number of 2.0. It is desired to find the condition of inlet operation that will yield maximum thrust minus drag or maximum range. For a given airplane with its engine, this also means determining the size of the inlet to be used on the airplane. It is assumed that the engine performance and performance derivatives are known or that it is satisfactory to use the data given in this report. In either case the engine air flow with respect to the airplane size must be known.

Using a break-even condition of this report, which is also the mathematical condition for a maximum, a value of $dC_D/d\mathscr{P}$ may be calculated. This drag coefficient is based on an engine area A_e. To apply this maximizing condition to the present problem, approximately

$$\frac{dC_D}{d\mathscr{P}} = \frac{dC_D}{d\left(\frac{m}{m_r}\right)} \frac{1}{\frac{d\mathscr{P}}{d\left(\frac{m}{m_r}\right)}} \tag{8a}$$

from which

$$\frac{d\mathscr{P}}{d\left(\frac{m}{m_r}\right)} = \frac{dC_{D,A_{max}}}{d\left(\frac{m}{m_r}\right)} \frac{1}{\frac{dC_D}{d\mathscr{P}} \frac{A_0/A_{max}}{A_0/A_e}} \tag{8b}$$

where $dC_{D,A_{max}}/d(m/m_r)$ is determined from the experimental data of figure 4, $dC_D/d\mathscr{P}$ is determined from the equations of this report, A_0/A_e is given in table III, and A_0/A_{max} must be known for the airplane under consideration at the pressure recovery given in table III.

The term A_0/A_{max} may be calculated, for example, when the actual compressor corrected air flow $(w_a\sqrt{\theta}/\delta)$ in pounds per second at the flight conditions under consideration and the airplane engine cross-sectional area in square feet corresponding to the model maximum cross-sectional area are known. For the usual case of $\gamma = 1.4$, $g = 32.2$, and $R = 53.3$,

$$\frac{A_0}{A_{max}} = \frac{1}{85.4} \frac{1}{A_{max}} \left(\frac{w_a\sqrt{\theta}}{\delta}\right)\mathscr{P} \frac{1}{\left(\frac{p}{P}\right)} \sqrt{\frac{t_s}{t}} \frac{1}{M} \tag{9}$$

Both p/P and t_s/t are functions of the flight Mach number M and tabulated in reference 4. The required value of $\mathscr{P}$ is given in table III.

For the present example, $A_0/A_{max} = 0.140$ and $dC_{D,A_{max}}/d(m/m_r) = -0.34$.

Using the values of $dC_D/d\mathscr{P}$ calculated in a previous example, the values of $d\mathscr{P}/d(m/m_r)$ for maximum thrust minus drag and maximum range are, respectively,

$$\frac{d\mathscr{P}}{d(m/m_r)} = -0.586$$

$$\frac{d\mathscr{P}}{d(m/m_r)} = -1.60$$

These slopes are plotted in figure 4. Where the curve of experimental pressure recovery against mass flow is tangent to these slopes, the thrust minus drag or range will be a maximum. The best inlet operating condition and size are thus determined for each of these conditions.

<u>Effect of flight Mach number and cruise cycle temperature on range importance of inlet drag and pressure recovery.</u> - To calculate the importance of pressure recovery on range, equation (B18) in table I may be
4043 simplified by setting dC_D and dW_e equal to zero, giving

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{d\mathcal{P}} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial \mathcal{P}}\right)_{t_{ab},C_D} - \left(\frac{\partial I}{\partial C_F}\right)_{\mathcal{P},C_D}\left(\frac{\partial C_F}{\partial \mathcal{P}}\right)_{t_{ab},C_D}\right\} \tag{10}$$

This equation has been evaluated for turbojet and ram-jet engines for two cycle temperatures over a range of flight Mach numbers. The results are presented in figure 5(a). The figure shows that, for a constant cycle temperature, the importance of a given increment in pressure recovery decreases with increasing flight Mach number. Also, pressure recovery is more important for airplanes designed to cruise at high cycle temperatures.

To calculate the range importance of the inlet drag, equation (B18) of table I becomes, taking X as C_D and letting dW_e be zero (where terms such as $(\partial I/\partial C_D)_{C_D}$ are zero),

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dC_{D,A_0}} = \frac{A_0}{A_e}\left\{-\frac{1}{C_F} + \frac{1}{I}\left(\frac{\partial I}{\partial C_F}\right)_{C_D}\right\} \tag{11}$$

where the inlet drag coefficient is based on the entering stream-tube area. The entering stream-tube area is the same as the area described by the inlet lip only for inlet operation with no spillage. The results of the calculation are shown in figure 5(b). The negative sign on the result simply means that increasing drag causes a reduction in range. The figure shows that the effect of an increment in drag coefficient on range increases as flight Mach number increases or as cruise cycle temperature is decreased.

The comparative importance of pressure recovery and drag may be calculated directly from equation (T3) of table I or from the previous results:

$$\left(\frac{dC_{D,A_0}}{d\mathcal{P}}\right)_{d\mathcal{R}=0} = -\frac{\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{d\mathcal{P}}}{\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dC_{D,A_0}}} \tag{12}$$

This is a "break-even" condition in that it relates the increment of drag coefficient that can be tolerated per unit increment in pressure recovery to result in zero net range change. (An increment of pressure recovery is the final value of pressure recovery minus the initial value.) The results of the calculation are shown in figure 5(c). The magnitude of the ordinate indicates the increase in drag coefficient that can be tolerated for a given increase in pressure recovery in order to maintain constant range. The figure shows that, at low Mach numbers, a large increase in drag coefficient is permissible for a given increase in pressure recovery. At high Mach numbers, only a small increase in drag coefficient is tolerable for the same increment in pressure recovery. At all Mach numbers, larger increases in drag coefficient are permissible (to maintain constant range) for the higher combustion temperature. Note that reference to constant range in connection with figure 5(c) means only that the range is held constant at each Mach number and at each combustor temperature as $\mathscr{P}$ and C_D are varied; it does not mean that the range is the same for all Mach numbers and combustor temperatures. 4043

The sensitivity of the present analysis to engine type and specific engine characteristics may be estimated from figure 5 by comparing the curves for the turbojet and ram-jet engines at 3500° R maximum cycle temperature. The ram-jet-powered airplane is more sensitive than the turbojet-powered airplane to both inlet drag and pressure recovery at flight Mach numbers below about 2.8. However, in the Mach number range between 2.0 and 3.0 where the ram-jet and turbojet curves overlap, the value of $dC_{D,A_O}/d\mathscr{P}$ is in general about the same for both engines. This indicates that the parameter $dC_{D,A_O}/d\mathscr{P}$ is not sensitive to engine type and, therefore, for example, should not be sensitive to differences that exist between various turbojet-engine designs.

CONCLUDING REMARKS

A simple method of evaluating propulsion-system components in terms of airplane range and thrust-minus-drag characteristics has been presented. Equations for evaluating changes in propulsion-system components both in existing airplanes and airplanes in the design stage were given. The method is particularly applicable for rapid evaluation of engine inlet pressure recovery, drag, and weight, and corresponding exhaust-nozzle characteristics. Representative engine cycle performance and performance derivatives for this purpose were also given.

Examples have been presented to illustrate typical applications of the method. For instance, one example showed that inlet pressure recovery becomes less important compared with nacelle drag as flight Mach number and combustion temperature are increased. Another example showed that pressure recovery is less important compared with nacelle drag when the criterion of airplane performance is range rather than acceleration potential.

Appendix D shows that, for airplanes cruising at the condition for maximum range, changes in range resulting from changes in engine component performance may be evaluated at either constant lift coefficient or constant engine combustor temperature. Also, the equations developed in this report in terms of engine performance parameters alone may alternatively be expressed in terms involving airplane aerodynamic parameters.

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Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
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APPENDIX A

SYMBOLS

A_e	engine reference area (compressor frontal area for turbojet, combustor flow area for ram jet)
A_l	cross-sectional area at inlet-cowl lip including centerbody area
A_{max}	model maximum cross-sectional area
A_0	free-stream tube area of air entering engine
C_D	engine component drag coefficient, D_{ex}/qA_e
$C_{D,i}$	coefficient of drag due to lift (based on S_w)
$C_{D,o}$	zero-lift drag coefficient of airplane (based on S_w)
C_F	engine internal thrust coefficient, F/qA_e
$C_{F,max}$	maximum C_F corresponding to $t_{ab,max}$
$C_{\mathcal{F}}$	engine propulsive thrust coefficient, $C_F - C_D$
$C_{\mathcal{F},max}$	maximum $C_{\mathcal{F}}$ corresponding to $t_{ab,max}$
C_L	airplane lift coefficient, L/qS_w
C_V	nozzle velocity coefficient
D_{ex}	external drag specifically associated with engine component being studied; considered to be included in D_t before component modification
dD_{ex}	change in external engine drag specifically associated with engine component modification being considered; equal to zero before modification
D_t	total airplane drag in steady level flight including all engine drag except dD_{ex}
F	engine internal thrust
$\mathcal{F}$	engine propulsive thrust, $F - D_{ex}$

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G	acceleration potential, $(\mathcal{F}_{max} - D_t)/W_g$
g	acceleration due to gravity
I	engine internal specific impulse, F/w_f
$\bar{I}$	engine specific impulse including drag, $\mathcal{F}/w_f$
k	$\left[1 - (W_f/W_g)\right] \ln\left[\frac{1 - (W_f/W_g)_\alpha}{1 - (W_f/W_g)}\right]$
L	airplane lift
M	flight Mach number
m/m_r	inlet mass-flow ratio, where m_r can be any reference mass flow
P	total pressure
P_N	total pressure at entrance to exhaust nozzle
$\mathcal{P}$	inlet diffuser pressure recovery, total pressure at diffuser exit divided by free-stream total pressure
p	ambient static pressure
q	free-stream incompressible dynamic pressure, $\frac{\gamma}{2} pM^2$
R	gas constant
$\mathcal{R}$	airplane range
S_w	wing area
t	total temperature
t_s	static temperature
V	flight speed
W_e	installed-engine weight
W_f	initial fuel weight
$W_{f,\alpha}$	fuel consumed during climb and acceleration
W_g	initial airplane gross weight

W_{pl} airplane fixed weight (payload, pilot, instruments, etc.)

W_{st} airplane structural weight (fuselage, wing, tail, fuel tank, landing gear, etc.)

w_a air-flow rate

w_f fuel-flow rate

X engine component parameter being considered

γ ratio of specific heats

δ ratio of total pressure to NACA standard sea-level pressure

θ ratio of total temperature to NACA standard sea-level temperature

Subscripts:

ab afterburner (or combustor)

exit nozzle exit

l cowl lip

max maximum

opt optimum

th nozzle throat

0 free-stream conditions

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APPENDIX B

DERIVATION OF EQUATIONS

Basic Considerations of Range Equation

The expressions for range that are derived in this report are based on the Breguet range equation. The integrated form of the range equation (eq. (1)) is obtained with the necessary condition that $V\bar{I}\ L/D_t$ is constant during the entire cruise phase of the flight. It is assumed in the present report that all cruising takes place in the isothermal atmosphere and at constant Mach number. In order to utilize the Breguet equation, $\bar{I}L/D_t$ must remain constant during flight, despite the fact that the airplane gross weight is constantly decreasing because of the consumption of fuel. The following discussion shows that both $\bar{I}$ and L/D_t may each be held constant during cruise flight.

The airplane lift-drag ratio is given by

$$\frac{L}{D_t} = \frac{C_L}{C_{D,o} + \left(\frac{C_{D,i}}{C_L^2}\right)C_L^2} = f(C_L) \qquad \text{(B1)}$$

where

$$C_L = \frac{W_g}{pS_W}\left(\frac{1}{\frac{\gamma}{2}\ M^2}\right) \qquad \text{(B2)}$$

The terms $C_{D,o}$ and $C_{D,i}/C_L^2$ are constants describing the airplane aerodynamics, so that L/D_t is a function only of C_L for all airplanes of the same geometric proportions, regardless of the actual size (neglecting Reynold's number effects). Mathematically, it is required that the term W_g/pS_W be constant in order that L/D_t not change for any given airplane during cruising; also, any two airplanes of similar proportions but unequal sizes or gross weights will have the same L/D_t if they cruise at the same value of W_g/pS_W.

A similar argument may be made with respect to the engine specific impulse. For a given engine flying at constant Mach number in the isothermal atmosphere, the specific impulse is a function only of the propulsive thrust coefficient (see fig. 2(a), e.g.):

$$\bar{I} = f(C_{\mathcal{F}}) \qquad \text{(B3)}$$

where

$$C_{\mathcal{F}} = \left(\frac{W_g}{pS_w}\right)\left(\frac{1}{\frac{L}{D_t}}\right)\left(\frac{1}{\frac{\gamma}{2}M^2}\right)\left(\frac{S_w}{A_e}\right) \tag{B4}$$

For a given airplane, S_w/A_e is constant and, from the preceding discussion, L/D_t does not vary if W_g/pS_w is held constant. Thus, $C_{\mathcal{F}}$, and hence $\bar{I}$, remains constant during cruising flight of a given airplane if W_g/pS_w is held constant. 4043

Consider now the flight path that must be selected to maintain constant W_g/pS_w. Assume a reference airplane that commences its cruise flight at a given Mach number and altitude represented by point A in figure 6(a). For this condition a specific value of W_g/pS_w exists. Throughout the flight, W_g continually decreases as fuel is consumed. To maintain the initial value of W_g/pS_w, it is necessary that p decrease in the same proportion; therefore, the altitude of the airplane increases from the beginning of its cruise flight A to the end B. This flight path, and thus constant values of W_g/pS_w, L/D_t, $C_{\mathcal{F}}$, and $\bar{I}$, is accomplished automatically by maintaining a constant engine combustion temperature throughout cruising flight.

Instead of starting cruise at the value of W_g/pS_w corresponding to point A, some other value of W_g/pS_w could have been selected, corresponding to point C, for example. This new value of W_g/pS_w would then be maintained along flight path C-D. In general, cruising at the new W_g/pS_w results in a different value of L/D_t during flight (since C_L must be changed to supply the needed lift) and in a different value of $\bar{I}$ during flight (since $C_{\mathcal{F}}$ must be changed to supply the needed thrust). From equation (1), to achieve maximum range, the value of W_g/pS_w should be selected to make $\bar{I}L/D_t$ a maximum. This is represented on figure 6(b) by point E.

A change in an engine component may require an adjustment in the cruising W_g/pS_w to remaximize the value of $\bar{I}L/D_t$. As shown by the figure, the change in $\bar{I}L/D_t$ resulting from an engine modification dX may be considered in two parts, the direct effect of dX at a constant

W_g/pS_w (from E to F) and the further effect of any accompanying change in cruising W_g/pS_w (from F to G). Mathematically,

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$$\bar{I}\,\frac{L}{D_t} = f\left(\frac{W_g}{pS_w}, X\right) \tag{B5}$$

$$\frac{d}{dX}\left(\bar{I}\,\frac{L}{D_t}\right) = \left[\frac{\partial}{\partial\left(\frac{W_g}{pS_w}\right)}\left(\bar{I}\,\frac{L}{D_t}\right)\right]_X \frac{d}{dX}\left(\frac{W_g}{pS_w}\right) + \left[\frac{\partial}{\partial X}\left(\bar{I}\,\frac{L}{D_t}\right)\right]_{\frac{W_g}{pS_w}} \tag{B6}$$

where the subscripts on the brackets indicate the quantity being held constant in the partial differentiation. If, with the original engine design, the airplane is initially cruising at point E (the point for maximum $\bar{I}L/D_t$), then

$$\left[\frac{\partial}{\partial\left(\frac{W_g}{pS_w}\right)}\left(\bar{I}\,\frac{L}{D_t}\right)\right]_X = 0 \tag{B7}$$

(Some further implications of eq. (B7) are discussed in appendix D.) Then, equation (B6) becomes

$$\frac{d}{dX}\left(\bar{I}\,\frac{L}{D_t}\right) = \left[\frac{\partial}{\partial X}\left(\bar{I}\,\frac{L}{D_t}\right)\right]_{\frac{W_g}{pS_w}} \tag{B8}$$

The significance of this result is that, for a small engine change dX, the corresponding change in $\bar{I}L/D_t$ may be calculated without considering any change in cruising W_g/pS_w.

Expanding equation (B8) gives

$$\frac{d}{dX}\left(\bar{I}\,\frac{L}{D_t}\right) = \frac{L}{D_t}\left(\frac{\partial \bar{I}}{\partial X}\right)_{\frac{W_g}{pS_w}} + \bar{I}\left[\frac{\partial}{\partial X}\left(\frac{L}{D_t}\right)\right]_{\frac{W_g}{pS_w}} \tag{B9}$$

But, from equations (B1) and (B2), the lift-drag ratio is a function only of cruising W_g/pS_w, so that

$$\left[\frac{\partial}{\partial X}\left(\frac{L}{D_t}\right)\right]_{\frac{W_g}{pS_w}} = 0 \tag{B10}$$

Combining equations (B9) and (B10) gives

$$\frac{d}{dX}\left(\bar{I}\,\frac{L}{D_t}\right) = \frac{L}{D_t}\left(\frac{\partial \bar{I}}{\partial X}\right)_{\frac{W_g}{pS_w}} \tag{B11}$$

Equations (2a) and (B11) may now be combined to give

$$\frac{1}{\mathcal{R}}\,\frac{d\mathcal{R}}{dX} = \frac{1}{\bar{I}}\,\frac{d\bar{I}}{dX} + \frac{1}{k}\,\frac{d}{dX}\left(\frac{W_f}{W_g}\right) \tag{2b}$$ 4043

For convenience, the partial-derivative symbol is dropped, with the understanding that the cruising W_g/pS_w and L/D_t are not variables in equation (2b) and all the succeeding equations.

Range of Fixed-Size Airplane

The preceding section concludes that W_g/pS_w is to be considered fixed when the engine is modified. In this section it is assumed that the airplane size and hence S_w are fixed; therefore, W_g/p is a constant. Assuming further that the engine size is not changed, differentiation of equation (B4) gives

$$\frac{dC_{\mathcal{F}}}{dX} = 0 \tag{B12}$$

because all the terms on the right side of the equation are constant.

The analysis to this point has dealt with the engine-airframe combination. Consider next the changes that take place in the engine itself. In general, a change in any engine parameter X tends to change both the thrust coefficient and the specific impulse of the engine. In order to keep the thrust coefficient unchanged, as required by equation (B12), it usually is necessary to adjust the combustion temperature of the cycle. (This will be the temperature of the afterburner if the engine has one; otherwise, it is the temperature in the primary combustor.) Adjustment in thrust may also occur through changes in the drag associated with the engine modification dX. In mathematical terms,

$$C_{\mathcal{F}} = f_1(X,\ C_D,\ t_{ab}) \tag{B13a}$$

$$\bar{I} = f_2(X,\ C_D,\ t_{ab}) \tag{B13b}$$

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By differentiation,

$$dC_{\mathcal{F}} = \left(\frac{\partial C_{\mathcal{F}}}{\partial X}\right)_{t_{ab},C_D} dX + \left(\frac{\partial C_{\mathcal{F}}}{\partial C_D}\right)_{t_{ab},X} dC_D + \left(\frac{\partial C_{\mathcal{F}}}{\partial t_{ab}}\right)_{X,C_D} dt_{ab} \qquad \text{(B14a)}$$

$$d\overline{I} = \left(\frac{\partial \overline{I}}{\partial X}\right)_{t_{ab},C_D} dX + \left(\frac{\partial \overline{I}}{\partial C_D}\right)_{t_{ab},X} dC_D + \left(\frac{\partial \overline{I}}{\partial t_{ab}}\right)_{X,C_D} dt_{ab} \qquad \text{(B14b)}$$

By definition,

$$C_{\mathcal{F}} = C_F - C_D \qquad \text{(B15a)}$$

$$\overline{I} = \left(\frac{C_{\mathcal{F}}}{w_f}\right) qA_e = \frac{C_{\mathcal{F}}}{C_F} I \qquad \text{(B15b)}$$

which may be differentiated to give

$$\left.\begin{aligned} \left(\frac{\partial C_{\mathcal{F}}}{\partial C_D}\right)_{t_{ab},X} &= -1 \\ \left(\frac{\partial C_{\mathcal{F}}}{\partial X}\right)_{C_D} &= \frac{\partial C_F}{\partial X} \end{aligned}\right\} \qquad \text{(B16a)}$$

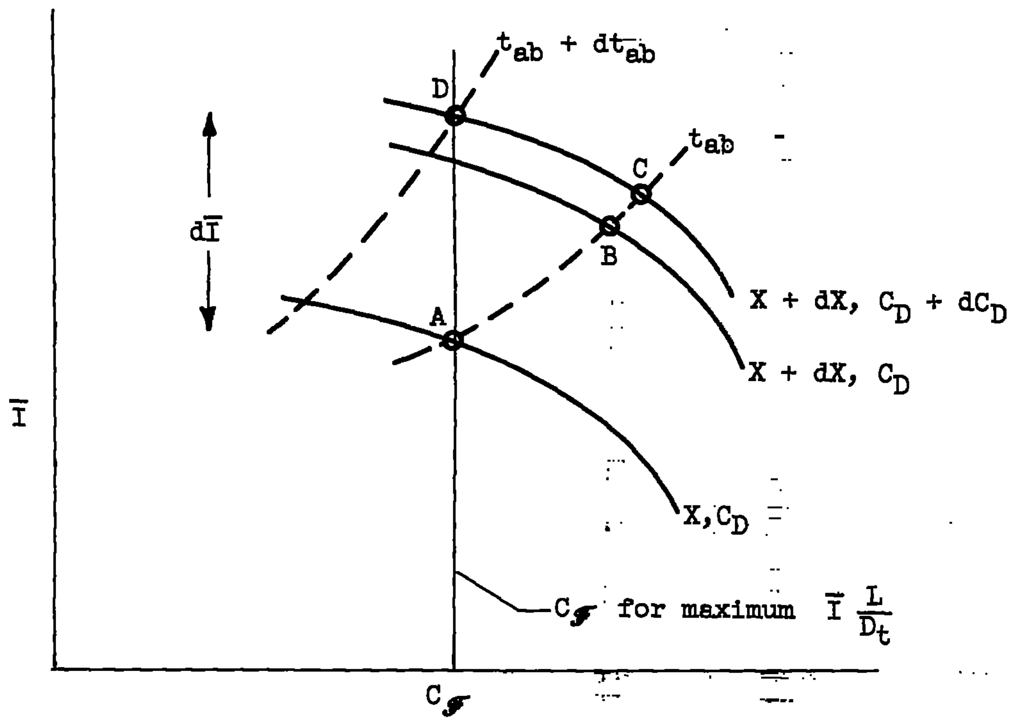
$$\left.\begin{aligned} \left(\frac{\partial \overline{I}}{\partial C_D}\right)_{t_{ab},X} &= -\frac{\overline{I}}{C_{\mathcal{F}}} \\ \left(\frac{\partial \overline{I}}{\partial X}\right)_{C_D} &= \frac{\partial I}{\partial X} \\ \left(\frac{\partial \overline{I}}{\partial C_{\mathcal{F}}}\right)_{X,C_D} &= \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D} \end{aligned}\right\} \qquad \text{(B16b)}$$

In addition, $C_{\mathcal{F}}$ and $\overline{I}$ for the reference or unmodified engine are equal to C_F and I, respectively, since the airplane lift-drag ratio is defined to include the original engine drag.

Combining equations (B12) and (B14) to (B16) gives

$$\frac{d\bar{I}}{dX} = \left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\right]\frac{dC_D}{dX} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} \tag{B17}$$

(The term $\partial I/\partial C_F$ is the ratio of the changes in I and C_F resulting from a small change in temperature dt_{ab}.) A graphical representation of these relations is as follows: 4043



The original airplane cruises with the engine operating at point A, which is at the thrust coefficient for maximum $\bar{I}L/D_t$. A change in an engine parameter dX changes the propulsive thrust coefficient and specific impulse to point B, if the cycle temperature is held constant. The associated variation in C_D, if any, causes a further change to point C. In order to return the thrust coefficient to its original value, as required by equation (B12), there must also be a change in cycle temperature dt_{ab}, so that the final engine operating condition is at point D.

Consider next the weight distribution of the airplane. The initial gross weight of an airplane is the sum of the various components, as follows:

$$W_g = W_{st} + W_{pl} + W_e + W_f \tag{3}$$

The effect of engine modifications on equation (3) is considered in two cases, the first assuming that W_g is held constant and the second allowing W_g to vary.

Constant initial gross weight. - For a fixed airframe and a constant initial gross weight, it is assumed that changes in engine weight can be accommodated only by a corresponding opposite change in the fuel load. Thus, differentiating equation (3) gives

$$\frac{dW_e}{dX} = -\frac{dW_f}{dX} \tag{4}$$

Combining equations (2b), (B17), and (4) gives the final general expression for the range of a fixed-size airplane with constant initial gross weight:

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\right]\frac{dC_D}{dX} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D}\right\} - \frac{1}{kW_g}\frac{dW_e}{dX} \tag{B18}$$

in which the partial derivatives are to be evaluated at the combustion temperature t_{ab} that yields maximum $\overline{I}L/D_t$ for the airplane. Equation (B18) indicates the effect on airplane range of any engine modification dX that also affects engine drag by an amount dC_D and engine weight by an amount dW_e. Setting $d\mathcal{R}$ equal to zero in equation (B18) results in the break-even condition for range. From the break-even equation it is possible to calculate the maximum permissible changes in engine drag and engine weight resulting from an engine modification such that the range does not change.

Equation (B18) may be further simplified for the cases in which either the weight or the drag changes, but not both. For example, if the engine modification affects only the engine weight, the break-even expression is obtained by setting $d\mathcal{R}$ and dC_D equal to zero in equation (B18), giving

$$\frac{1}{W_g}\frac{dW_e}{dX} = \frac{k}{I}\left[\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D}\right]\left\{\begin{array}{l} d\mathcal{R} = 0 \\ dW_g = 0 \\ dC_D = 0 \end{array}\right. \tag{B19a}$$

while setting $d\mathcal{R}$ and dW_e equal to zero yields

$$\frac{dC_D}{dX} = \frac{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D}}{\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}} \quad \begin{cases} d\mathcal{R} = 0 \\ dW_g = 0 \\ dW_e = 0 \end{cases} \tag{B19b}$$

Equations (B19) may be simplified still further for the special case in which the engine weight is a negligible part of the airplane gross weight. This condition might be approximated in the case of a ram-jet-powered long-range missile, for example. If the engine weight is negligible, there is no advantage in raising the thrust in order to reduce engine size. The engine can then be sized to cruise at the low temperature yielding maximum specific impulse. Setting $(\partial I/\partial C_F)_{X,C_D}$ equal to zero in equations (B19a) and (B19b), respectively, gives

$$\frac{1}{W_g}\frac{dW_e}{dX} = \frac{k}{I}\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} \quad \begin{cases} d\mathcal{R} = 0 \\ dW_g = 0 \\ dC_D = 0 \\ W_e \ll W_g \end{cases} \tag{B20a}$$

$$\frac{dC_D}{dX} = \frac{C_F}{I}\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} \quad \begin{cases} d\mathcal{R} = 0 \\ dW_g = 0 \\ dW_e = 0 \\ W_e \ll W_g \end{cases} \tag{B20b}$$

Another simplification arises when the parameter X relates to the exhaust nozzle (e.g., when X is the nozzle velocity coefficient C_V). The efficiency of expansion through the exhaust nozzle does not affect the engine air flow nor the fuel-air ratio. Therefore, the engine thrust coefficient and the specific impulse change in the same proportion when C_V is varied with constant combustion temperature. That is,

$$I \propto C_F$$

$$\left(\frac{\partial I}{\partial C_V}\right)_{t_{ab},C_D} = \frac{I}{C_F}\left(\frac{\partial C_F}{\partial C_V}\right)_{t_{ab},C_D} \tag{B21}$$

Substituting this expression in equations (B19a) and (B19b) gives

$$\frac{1}{W_g}\frac{dW_e}{dC_V} = \frac{k}{I}\left(\frac{\partial C_F}{\partial C_V}\right)_{t_{ab},C_D}\left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{C_V,C_D}\right] \begin{cases} d\mathcal{R} = 0 \\ dW_g = 0 \\ dC_D = 0 \end{cases} \qquad \text{(B22a)}$$

$$\frac{dC_D}{dC_V} = \left(\frac{\partial C_F}{\partial C_V}\right)_{t_{ab},C_D} \begin{cases} d\mathcal{R} = 0 \\ dW_g = 0 \\ dW_e = 0 \end{cases} \qquad \text{(B22b)}$$

<u>Variable initial gross weight</u>. - The preceding section has considered an airplane of fixed size and constant initial gross weight. When an engine change was made that resulted in an increase in engine weight, it was assumed that the fuel tanks could not be completely filled. Another case of practical interest occurs when the initial gross weight is permitted to increase by keeping a full fuel load even when the engine weight is increased. The airplane thus is overloaded during takeoff, but this may often be an acceptable penalty in order to achieve maximum range. Differentiating equation (3) with a constant fuel load gives

$$\frac{dW_g}{dX} = \frac{dW_e}{dX} \qquad (5)$$

The change in ratio of fuel weight to gross weight due to a change in gross weight for a fixed fuel weight is, by differentiation and using equation (5),

$$\frac{d}{dX}\left(\frac{W_f}{W_g}\right) = -\left(\frac{1}{W_g}\right)\left(\frac{W_f}{W_g}\right)\frac{dW_e}{dX} \qquad \text{(B23)}$$

Combining equations (2b), (B17), and (B23) gives

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\right]\frac{dC_D}{dX} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D}\right\} - \left(\frac{1}{kW_g}\right)\left(\frac{W_f}{W_g}\right)\frac{dW_e}{dX} \qquad \text{(B24)}$$

Simplified equations similar to those derived for the case of constant initial gross weight (eqs. (B19) to (B22)) are easily obtained.

Acceleration Potential of Fixed-Size Airplane

The margin of engine thrust over airplane drag is important to the amount of fuel consumed during acceleration and climb, the time to accelerate and climb, the ceiling, and the combat maneuverability. The margin of maximum engine thrust over cruising airplane drag is expressed in the present report in terms of the acceleration potential G, defined as

$$G = \frac{\mathcal{F}_{max} - D_t}{W_g} = \left(\frac{C_{\mathcal{F},max} A_e}{W_g}\right) q - \frac{1}{\frac{L}{D_t}} \tag{6a}$$

where the subscript max means that $C_{\mathcal{F}}$ is to be evaluated at the maximum allowable combustion temperature for highest thrust. From the definition of $C_{\mathcal{F}}$, equation (6a) may be written as

$$G = \left(C_{F,max} - C_D\right)\left(\frac{A_e}{W_g}\right) q - \frac{1}{\frac{L}{D_t}} \tag{B25}$$

The effect of an engine modification on the thrust margin is obtained by differentiating equation (B25), giving

$$\frac{dG}{dX} = \left[\frac{dC_{F,max}}{dX} - \frac{dC_D}{dX} - \left(\frac{C_{F,max} - C_D}{W_g}\right)\frac{dW_g}{dX}\right]\left(\frac{A_e}{W_g}\right) q \tag{B26}$$

In this section, as previously, advantage is taken of the fact that, for the unmodified engine, C_D equals zero because of the definition of L/D_t, and hence $C_{\mathcal{F}}$ is equal to C_F. Thus, combining equations (6a) and (B26) and using the relation $C_F q A_e = W_g/(L/D_t)$ give

$$\frac{dG}{dX} = \left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{C_D} - \frac{dC_D}{dX} - \frac{C_{F,max}}{W_g}\frac{dW_g}{dX}\right]\left(\frac{1}{C_F}\right)\left(\frac{1}{\frac{L}{D_t}}\right) \tag{B27}$$

and the break-even condition $(dG/dX = 0)$ is

$$\frac{dC_D}{dX} = \left(\frac{\partial C_{F,max}}{\partial X}\right)_{C_D} - \frac{C_{F,max}}{W_g}\frac{dW_g}{dX} \tag{B28}$$

Note that the break-even condition is also the condition for maximum acceleration potential and is independent of L/D_t and C_F. For a fixed-size airplane with constant gross weight, dW_g/dX is taken equal to zero. For a fixed-size airplane with variable gross weight, dW_g/dX is taken equal to dW_e/dX (according to eq. (5)).

Range of Variable-Size Airplane with

Constant Acceleration Potential

In this section the geometrical proportions of the airframe are fixed, but it is assumed that the size may be varied. This allows imposing the condition independently of any variations in airplane range that the acceleration potential does not change as the engine is modified. This situation may exist during the preliminary design stage of the airplane. The equations are developed for the case of constant engine size, although they can easily be expressed in terms of the ratio A_e/W_g, which then permits varying the engine size if desired.

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Consider first the engine-airframe combination. As discussed in the first part of this appendix, assume that the airplane is always designed for the same value of the term W_g/pS_w. Since the airframe proportions are specified, the cruising L/D_t does not change and equation (2b) applies.

For cruising flight the thrust must equal the drag, so that the criterion for constant acceleration potential can be written

$$\frac{d}{dX}\left(\frac{C_{\mathcal{F},max} - C_{\mathcal{F}}}{W_g}\right) = 0 \qquad \text{(B29)}$$

Carrying out the differentiation gives

$$\frac{C_{\mathcal{F},max}}{W_g}\frac{d}{dX}\left(1 - \frac{C_{\mathcal{F}}}{C_{\mathcal{F},max}}\right) + \left(1 - \frac{C_{\mathcal{F}}}{C_{\mathcal{F},max}}\right)\frac{d}{dX}\left(\frac{C_{\mathcal{F},max}}{W_g}\right) = 0 \qquad \text{(B30)}$$

Also, differentiating equation (6a) gives

$$\frac{dG}{dX} = A_e\, q\, \frac{d}{dX}\left(\frac{C_{\mathcal{F},max}}{W_g}\right) \qquad \text{(B31)}$$

which, for a constant maneuver margin $(dG/dX = 0)$, reduces to

$$\frac{d}{dX}\left(\frac{C_{\mathcal{F},max}}{W_g}\right) = 0 \qquad \text{(B32)}$$

Combining equations (B30) and (B32) gives

$$\frac{d}{dX}\left(\frac{C_{\mathcal{F}}}{C_{\mathcal{F},max}}\right) = 0 \qquad \text{(B33)}$$

Now examine the engine more closely. The changes that occur in the engine are defined by carrying out the differentiation indicated by equation (B33), resulting in

$$\frac{dC_{\mathcal{F}}}{dX} = \frac{C_{\mathcal{F}}}{C_{\mathcal{F},max}} \frac{dC_{\mathcal{F},max}}{dX} \tag{B34}$$

From equation (B14a) evaluated at the temperature for maximum thrust,

$$\frac{dC_{\mathcal{F},max}}{dX} = \left(\frac{\partial C_{\mathcal{F},max}}{\partial X}\right)_{t_{ab},C_D} + \left(\frac{\partial C_{\mathcal{F},max}}{\partial C_D}\right)_{t_{ab},X} \frac{dC_D}{dX} + \left(\frac{\partial C_{\mathcal{F},max}}{\partial t_{ab}}\right)_{X,C_D} \frac{dt_{ab,max}}{dX} \tag{B35}$$

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Maximum thrust of the engine is ordinarily obtained at the maximum permissible cycle temperature, so that the third term on the right side of equation (B35) differs from zero only if the engine modification dX results in a change in maximum permissible combustion temperature $t_{ab,max}$.

The change in engine specific impulse resulting from an engine modification is found by combining equations (B14) to (B16), (B34), and (B35), giving

$$\frac{dI}{dX} = \left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX}\left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(1 - \frac{C_F}{C_{F,max}}\right)\right] - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left[\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} - \frac{C_F}{C_{F,max}}\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{C_F}{C_{F,max}}\left(\frac{\partial C_{F,max}}{\partial t_{ab}}\right)_{X,C_D} \frac{dt_{ab,max}}{dX}\right] \tag{B36}$$

As before, $C_{\mathcal{F}}$ is equal to C_F before modification.

The change in weight distribution of the airplane is found by differentiating equation (3) with the assumption that the ratio of structural to gross weight does not change with X:

$$\frac{d}{dX}\left(\frac{W_{pl}}{W_g}\right) + \frac{d}{dX}\left(\frac{W_e}{W_g}\right) + \frac{d}{dX}\left(\frac{W_f}{W_g}\right) = 0 \tag{B37}$$

Also,

$$\frac{dW_e}{dX} = \frac{W_e}{W_g}\frac{dW_g}{dX} + W_g \frac{d}{dX}\left(\frac{W_e}{W_g}\right) \tag{B38}$$

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while expanding equation (B32) gives

$$\frac{dC_{F,max}}{dX} = \frac{C_{F,max}}{W_g}\frac{dW_g}{dX} \tag{B39}$$

Combining equations (B16), (B35), (B38), and (B39) gives

$$\frac{d}{dX}\left(\frac{W_f}{W_g}\right) = -\frac{d}{dX}\left(\frac{W_{pl}}{W_g}\right) - \frac{1}{W_g}\frac{dW_e}{dX} + \frac{1}{C_{F,max}}\frac{W_e}{W_g}$$

$$\left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX} + \left(\frac{\partial C_{F,max}}{\partial t_{ab}}\right)_{X,C_D}\frac{dt_{ab,max}}{dX}\right] \tag{B40}$$

Substituting equations (B36) and (B40) into equation (2b) then gives the general result for a variable-size airplane:

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(1 - \frac{C_F}{C_{F,max}}\right)\right]\frac{dC_D}{dX} - \right.$$

$$\left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left[\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} - \frac{C_F}{C_{F,max}}\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \right.$$

$$\left.\left.\frac{C_F}{C_{F,max}}\left(\frac{\partial C_{F,max}}{\partial t_{ab}}\right)_{X,C_D}\frac{dt_{ab,max}}{dX}\right]\right\} - \frac{1}{k}\left\{\frac{d}{dX}\left(\frac{W_{pl}}{W_g}\right) + \frac{1}{W_g}\frac{dW_e}{dX} - \right.$$

$$\left.\frac{1}{C_{F,max}}\left(\frac{W_e}{W_g}\right)\left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX} + \left(\frac{\partial C_{F,max}}{\partial t_{ab}}\right)_{X,C_D}\frac{dt_{ab,max}}{dX}\right]\right\} \tag{B41}$$

Two points of view may be taken in considering the ratio of payload to gross weight W_{pl}/W_g. One approach is to assume that the payload remains constant when the airplane gross weight varies. This gives

$$\frac{d}{dX}\left(\frac{W_{pl}}{W_g}\right) = -\frac{1}{W_g}\left(\frac{W_{pl}}{W_g}\right)\frac{dW_g}{dX} \tag{B42}$$

which combines with equation (B39) to give

$$\frac{d}{dX}\left(\frac{W_{pl}}{W_g}\right) = -\left(\frac{W_{pl}}{W_g}\right)\left(\frac{1}{C_{F,max}}\right)\frac{dC_{F,max}}{dX} \tag{B43}$$

Combining equations (B16), (B35), and (B43) gives

$$\frac{d}{dX}\left(\frac{W_{pl}}{W_g}\right) = -\left(\frac{W_{pl}}{W_g}\right)\left(\frac{1}{C_{F,max}}\right)\left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX} + \left(\frac{\partial C_{F,max}}{\partial t_{ab}}\right)_{X,C_D}\frac{dt_{ab,max}}{dX}\right] \tag{B44}$$

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Substituting this expression in equation (B41) gives the result for the case of a constant payload:

$$\begin{aligned}\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{I}\Bigg\{&\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(1 - \frac{C_F}{C_{F,max}}\right)\right]\frac{dC_D}{dX} - \\ &\left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left[\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} - \frac{C_F}{C_{F,max}}\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \right. \\ &\left.\frac{C_F}{C_{F,max}}\left(\frac{\partial C_{F,max}}{\partial t_{ab}}\right)_{X,C_D}\frac{dt_{ab,max}}{dX}\right]\Bigg\} - \frac{1}{k}\Bigg\{\frac{1}{W_g}\frac{dW_e}{dX} - \\ &\frac{1}{C_{F,max}}\left(\frac{W_e}{W_g} + \frac{W_{pl}}{W_g}\right)\left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX} + \right. \\ &\left.\left(\frac{\partial C_{F,max}}{\partial t_{ab}}\right)_{X,C_D}\frac{dt_{ab,max}}{dX}\right]\Bigg\}\end{aligned} \tag{B45}$$

A second point of view is to assume that the ratio of payload to gross weight remains constant when the airplane gross weight varies. This case is obtained simply by taking $d(W_{pl}/W_g)/dX$ equal to zero in equation (B41).

APPENDIX C

ASSUMPTIONS FOR CALCULATING ENGINE PERFORMANCE AND DERIVATIVES

The component assumptions made for the turbojet engine are:

(1) Inlet diffuser (see table III)

(2) Compressor:

Sea-level static pressure ratio, 7
Sea-level static air flow, 35(lb/sec)/sq ft
Constant mechanical speed

(3) Combustor:

Fuel, JP-4
Primary burner efficiency, 0.95
Afterburner efficiency, 0.90

(4) Turbine-inlet temperature, 2500° R

(5) Exhaust-nozzle force coefficient, 0.96

The component assumptions made for the ram-jet engine are:

(1) Inlet diffuser (see table III)

(2) Combustor:

Inlet Mach number, 0.175
Fuel, JP-4
Efficiency, 0.90

(3) Exhaust-nozzle force coefficient, 0.96

The thrust coefficient for the turbojet engine is based on the compressor tip area. This is reasonable, since compressor tip area is not likely to be varied as a result of engine modifications. It is for this reason that the equations were developed for the case of constant engine (i.e., compressor tip) area.

When the derivatives of $C_{\mathcal{F}}$ and I with respect to inlet pressure recovery were calculated, it was assumed that the compressor-inlet Mach number (i.e., diffuser-exit Mach number) was held constant. For supersonic flight, afterburning is generally required. Best efficiency is then obtained by maintaining rated turbine-inlet temperature and varying the thrust output as needed through changes in afterburner temperature and concomitant nozzle-throat area changes. For a constant turbine-inlet temperature, the rotative speed and corrected air flow of the compressor

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are also constant (at any given flight Mach number and ambient temperature). Constant corrected compressor air flow is equivalent to a constant compressor-inlet Mach number.

As a consequence of these assumptions, the air flow through the engine and the ratio of inlet lip to compressor tip area A_l/A_e vary directly with the inlet pressure recovery (provided the inlet operates at the same mass-flow ratio for both recoveries). If the maximum pressure recovery of an inlet is improved through some physical modification but the lip area is not increased, then the inlet will operate supercritically (i.e., the normal shock will move downstream into the subsonic diffuser), and the actual operating pressure recovery of the inlet will be no greater than before, despite the potentially available increase. Conversely, if the effect of the modification is to decrease the pressure recovery, then the air flow through the engine is also decreased; if the lip area is not reduced, the excess air is spilled around the inlet and results in additive drag, which further penalizes the engine thrust and efficiency. (Note that, for a constant compressor-inlet Mach number and combustor temperature, the exhaust-nozzle-throat area is independent of changes in inlet pressure recovery.)

The preceding discussion shows that, for the turbojet engine, the diffuser-exit Mach number is not variable and the inlet area should be varied when pressure recovery is changed.

For the ram-jet engine, the thrust coefficient is based on the combustor frontal area, and constant diffuser-exit Mach number was also assumed when $\partial C_F/\partial \mathcal{P}$ and $\partial I/\partial \mathcal{P}$ were calculated. (As in the case of the turbojet, this means that the exhaust-nozzle-throat area is independent of changes in inlet pressure recovery.) However, other assumptions can be made about the diffuser-exit Mach number. In this case, there is no compressor with its requirement of constant corrected air flow. It is feasible then, if desired, to assume that the diffuser area ratio A_l/A_e is held fixed, so that the combustor-inlet Mach number varies with changes in diffuser pressure recovery. (Note that, for a given combustion temperature, this requires some adjustment in exhaust-nozzle-throat area.) For a constant combustion temperature, and neglecting the small variation with inlet Mach number of the combustor momentum-pressure loss due to heat addition, the following relation holds:

$$\frac{1}{C_F}\left(\frac{\partial C_F}{\partial \mathcal{P}}\right)_{t_{ab},C_D} = \frac{1}{I}\left(\frac{\partial I}{\partial \mathcal{P}}\right)_{t_{ab},C_D} \tag{C1}$$

The equations of tables I and II are still valid, and equation (C1) may be used to evaluate $\partial C_F/\partial \mathcal{P}$ for the ram-jet engine where A_l/A_e is held constant ($\partial I/\partial \mathcal{P}$ being taken from figure 2(d)).

APPENDIX D

ALTERNATIVE DEVELOPMENT OF METHOD

The equations of the present report were derived under the basic assumption that the value of W_g/pS_w is held constant when an engine modification dX is made. By inference, any required adjustment in engine thrust is then made by changing the engine combustion temperature. This appendix proves that the engine may, if desired, be viewed as continuing to cruise at the same temperature, with the required adjustment in engine thrust made by changing airplane altitude. The same final expressions for range are obtained in either case.

This appendix also shows that the equations of the present report, which are expressed in terms of the engine parameters, can alternatively be expressed in terms of the airplane aerodynamic parameters.

Demonstration of Alternative Viewpoint

Identical results are obtained regardless of whether the airplane flies at constant W_g/pS_w or the engine operates at constant temperature. To prove this statement it must be demonstrated that

$$\left[\frac{D\left(\frac{IL}{D_t}\right)}{\partial X}\right]_{t_{ab}} = \left[\frac{D\left(\frac{IL}{D_t}\right)}{\partial X}\right]_{\frac{W_g}{pS_w}} \tag{D1}$$

Expanding the right side of equation (D1) gives

$$\left[\frac{D\left(\frac{IL}{D_t}\right)}{\partial X}\right]_{\frac{W_g}{pS_w}} = I\left[\frac{D\left(\frac{L}{D_t}\right)}{\partial X}\right]_{\frac{W_g}{pS_w}} + \frac{L}{D_t}\left(\frac{DI}{\partial X}\right)_{\frac{W_g}{pS_w}} \tag{D2}$$

Substituting equations (B10) and (B17) into equation (D2) yields

$$\left[\frac{D\left(\frac{IL}{D_t}\right)}{\partial X}\right]_{\frac{W_g}{pS_w}} = \frac{L}{D_t}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \frac{I}{C_F}\frac{dC_D}{dX} + \left(\frac{\partial I}{\partial C_F}\right)_{C_D,X}\left[-\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} + \frac{dC_D}{dX}\right]\right\} \tag{D3}$$

where only a fixed-size airplane is considered.

Consider now the left side of equation (D1). Expanding,

$$\left[\frac{D\left(\frac{IL}{D_t}\right)}{\partial X}\right]_{t_{ab}} = I\left[\frac{D\left(\frac{L}{D_t}\right)}{\partial X}\right]_{t_{ab}} + \frac{L}{D_t}\left(\frac{DI}{\partial X}\right)_{t_{ab}} \tag{D4}$$

From equations (B14b) and (B16b),

$$\left(\frac{DI}{\partial X}\right)_{t_{ab}} = \left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \frac{I}{C_F}\frac{dC_D}{dX} \tag{D5}$$

Now,

$$\left[\frac{D\left(\frac{L}{D_t}\right)}{\partial X}\right]_{t_{ab}} = \frac{d\left(\frac{L}{D_t}\right)}{d\left(\frac{W_g}{pS_w}\right)}\left[\frac{D\left(\frac{W_g}{pS_w}\right)}{\partial X}\right]_{t_{ab}} \tag{D6}$$

Also, expanding equation (B7),

$$\left[\frac{D\left(\frac{IL}{D_t}\right)}{\partial\left(\frac{W_g}{pS_w}\right)}\right]_X = I\frac{d\left(\frac{L}{D_t}\right)}{d\left(\frac{W_g}{pS_w}\right)} + \frac{L}{D_t}\left[\frac{DI}{\partial\left(\frac{W_g}{pS_w}\right)}\right]_X = 0 \tag{D7}$$

The last term of equation (D7) may be rewritten

$$\left[\frac{DI}{\partial\left(\frac{W_g}{pS_w}\right)}\right]_X = \left(\frac{\partial I}{\partial C_F}\right)_X \frac{dC_F}{d\left(\frac{W_g}{pS_w}\right)} \tag{D8}$$

and differentiating equation (B4) gives

$$\left(\frac{DC_F}{\partial X}\right)_{t_{ab}} = \frac{dC_F}{d\left(\frac{W_g}{pS_w}\right)}\left[\frac{D\left(\frac{W_g}{pS_w}\right)}{\partial X}\right]_{t_{ab}} \tag{D9}$$

Equations (B14a) and (B16a) give

$$\left(\frac{DC_F}{\partial X}\right)_{t_{ab}} = \left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX} \tag{D10}$$

Combining equations (D6) to (D10) gives

$$\left[\frac{D\left(\frac{L}{D_t}\right)}{\partial X}\right]_{t_{ab}} = -\frac{1}{I}\left(\frac{L}{D_t}\right)\left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left[\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX}\right] \qquad \text{(D11)}$$

Equations (D5) and (D11) may now be substituted in equation (D4), yielding

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$$\left[\frac{D\left(\frac{IL}{D_t}\right)}{\partial X}\right]_{t_{ab}} = \frac{L}{D_t}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \frac{I}{C_F}\frac{dC_D}{dX} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left[\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX}\right]\right\} \qquad \text{(D12)}$$

The right sides of equations (D3) and (D12) are identical; hence equation (D1) has been proved.

When temperature is held constant (and W_g/pS_w allowed to vary), part of the range gain due to an engine component improvement dX results from an increase in I and part from an increase in airplane L/D_t. When W_g/pS_w is held constant (and temperature allowed to vary), all the range gain results from increases in I. Both cases yield the same answer.

The preceding discussion has assumed a constant airplane size. With the aid of some simplifying assumptions, the same result may be obtained when airplane size is allowed to vary.

Development in Terms of Aerodynamic Parameters

The equations in the present report are presented in terms of the engine performance parameters and their derivatives. The equations can alternatively be presented in terms of the airplane aerodynamic parameters. This latter approach, which was used in reference 3, is explained as follows.

The condition that the airplane initially cruises at maximum IL/D_t is given in equation (B7) and rewritten in equation (D7) as follows:

$$I\,\frac{d\left(\frac{L}{D_t}\right)}{d\left(\frac{W_g}{pS_w}\right)} + \frac{L}{D_t}\left[\frac{DI}{\partial\left(\frac{W_g}{pS_w}\right)}\right]_X = 0 \qquad \text{(D7)}$$

Considering the term on the right, write

$$\left[\frac{DI}{\partial\left(\frac{W_g}{pS_w}\right)}\right]_X = \left(\frac{\partial I}{\partial C_{\mathcal{F}}}\right)_{X,C_D}\left[\frac{DC_{\mathcal{F}}}{\partial\left(\frac{W_g}{pS_w}\right)}\right]_X \tag{D13}$$

where differentiating equation (B4) gives

$$\left[\frac{DC_{\mathcal{F}}}{\partial\left(\frac{W_g}{pS_w}\right)}\right]_X = C_{\mathcal{F}}\left[\frac{1}{\left(\frac{W_g}{pS_w}\right)} - \frac{1}{\frac{L}{D_t}}\,\frac{d\left(\frac{L}{D_t}\right)}{d\left(\frac{W_g}{pS_w}\right)}\right] \tag{D14}$$

Consider next the first term in equation (D7), which now also appears in equation (D14). Differentiating equation (B1) gives

$$\frac{1}{\frac{L}{D_t}}\,\frac{d\left(\frac{L}{D_t}\right)}{d\left(\frac{W_g}{pS_w}\right)} = \frac{dC_L}{d\left(\frac{W_g}{pS_w}\right)}\left[\frac{1}{C_L} - \frac{2\left(\frac{C_{D,i}}{C_L^2}\right)C_L}{C_{D,o} + \left(\frac{C_{D,i}}{C_L^2}\right)C_L^2}\right] \tag{D15}$$

where, from equation (B2),

$$\frac{1}{C_L}\,\frac{dC_L}{d\left(\frac{W_g}{pS_w}\right)} = \frac{1}{\left(\frac{W_g}{pS_w}\right)} \tag{D16}$$

hence, equation (D15) becomes

$$\frac{1}{\frac{L}{D_t}}\,\frac{d\left(\frac{L}{D_t}\right)}{d\left(\frac{W_g}{pS_w}\right)} = \frac{1}{\left(\frac{W_g}{pS_w}\right)}\left[\frac{1 - \left(\frac{C_L^2}{C_{D,o}}\right)\left(\frac{C_{D,i}}{C_L^2}\right)}{1 + \left(\frac{C_L^2}{C_{D,o}}\right)\left(\frac{C_{D,i}}{C_L^2}\right)}\right] \tag{D17}$$

Differentiation of equation (B1) with respect to C_L shows that the optimum value of C_L corresponding to maximum L/D_t is given by

$$C_{L,opt} = \sqrt{\frac{C_{D,o}}{\frac{C_{D,i}}{C_L^2}}} \tag{D18}$$

Combining equations (D7), (D14), (D17), and (D18) gives

$$\frac{C_F}{I}\left(\frac{\partial I}{\partial C_F}\right)_{X,C_D} = -\left[\frac{1 - \left(\frac{C_L}{C_{L,opt}}\right)^2}{2\left(\frac{C_L}{C_{L,opt}}\right)^2}\right] \qquad \text{(D19)}$$

4043 This expression shows that selecting a combustion temperature at which to evaluate the engine parameters necessarily implies that the airplane is cruising at a particular value of $C_L/C_{L,opt}$.

Equation (D19) may be used to eliminate the term $\partial I/\partial C_F$ from the equations presented in this report. Other engine parameters will still be present, however, such as C_F and I. It thus becomes possible to select a value of $C_L/C_{L,opt}$ for cruising, rather than an engine combustion temperature. Through equation (D19), the selected value of $C_L/C_{L,opt}$ then fixes the combustion temperature at which the remaining engine parameters should be evaluated (if the airplane cruises at maximum IL/D_t).

Although it is always assumed in the development of the equations of the present report that cruising occurs at maximum IL/D_t, other cases exist. For example, engine thrust may be limited so that, even at maximum combustion temperature, cruising takes place at altitudes lower than those for maximum IL/D_t. In this case it is desirable to write the equations in terms of $C_L/C_{L,opt}$ (using eq. (D19)), where the remaining engine parameters are evaluated for the maximum permissible temperature and $C_L/C_{L,opt}$ is determined from the airframe design and operating condition. From the statement of the problem, this value of $C_L/C_{L,opt}$ when substituted in equation (D19) must correspond to a value of $\frac{C_F}{I}\left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}$ for some higher than maximum permissible combustion temperature. This is the case considered in reference 3, which derived equations similar to those of the present report with $\partial I/\partial C_F$ replaced by the function of $C_L/C_{L,opt}$ given in equation (D19).

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TABLE I. - RANGE AND WEIGHT EXPRESSIONS

[Evaluate at cruise combustor temperature except terms involving $C_{F,max}$]

(a) Fixed-size airplane with constant initial gross weight

General equations:

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\right]\frac{dC_D}{dX} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D}\right\} - \frac{1}{kW_g}\frac{dW_e}{dX} \tag{B18}$$

$$\frac{dW_g}{dX} = 0 \tag{T1}$$

Illustrative special equations:

Effect of inlet pressure recovery and weight (constant inlet drag):

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{d\mathcal{P}} = \frac{1}{I}\left[\left(\frac{\partial I}{\partial \mathcal{P}}\right)_{t_{ab},C_D} - \left(\frac{\partial I}{\partial C_F}\right)_{\mathcal{P},C_D}\left(\frac{\partial C_F}{\partial \mathcal{P}}\right)_{t_{ab},C_D}\right] - \frac{1}{kW_g}\frac{dW_e}{d\mathcal{P}} \tag{T2}$$

Effect of inlet pressure recovery and drag (constant inlet weight):

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{d\mathcal{P}} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial \mathcal{P}}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{\mathcal{P},C_D}\right]\frac{dC_D}{d\mathcal{P}} - \left(\frac{\partial I}{\partial C_F}\right)_{\mathcal{P},C_D}\left(\frac{\partial C_F}{\partial \mathcal{P}}\right)_{t_{ab},C_D}\right\} \tag{T3}$$

Effect of exhaust-nozzle velocity coefficient and weight (constant nozzle drag):

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dC_V} = \frac{1}{I}\left(\frac{\partial C_F}{\partial C_V}\right)_{t_{ab},C_D}\left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{C_V,C_D}\right] - \frac{1}{kW_g}\frac{dW_e}{dC_V} \tag{T4}$$

Effect of exhaust-nozzle velocity coefficient and drag (constant nozzle weight):

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dC_V} = \frac{1}{I}\left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{C_V,C_D}\right]\left[\left(\frac{\partial C_F}{\partial C_V}\right)_{t_{ab},C_D} - \frac{dC_D}{dC_V}\right] \tag{T5}$$

TABLE I. - Continued. RANGE AND WEIGHT EXPRESSIONS

[Evaluate at cruise combustor temperature except terms involving $C_{F,max}$.]

(b) Fixed-size airplane with fixed initial fuel weight and variable initial gross weight

General equations:

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\right]\frac{dC_D}{dX} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D}\right\} -$$

$$\left(\frac{1}{kW_g}\right)\left(\frac{W_f}{W_g}\right)\frac{dW_e}{dX} \qquad \text{(B24)}$$

$$\frac{dW_g}{dX} = \frac{dW_e}{dX} \qquad (5)$$

TABLE I. - Concluded. RANGE AND WEIGHT EXPRESSIONS

[Evaluate at cruise combustor temperature except terms involving $C_{F,max}$.]

(c) Variable-size airplane with constant acceleration potential, $\frac{dt_{ab,max}}{dX} = 0$

General equations:

Constant payload weight:

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(1 - \frac{C_F}{C_{F,max}}\right)\right]\frac{dC_D}{dX} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left[\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} - \frac{C_F}{C_{F,max}}\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D}\right]\right\} - \frac{1}{k}\left\{\frac{1}{W_g}\frac{dW_e}{dX} - \frac{1}{C_{F,max}}\left(\frac{W_e}{W_g} + \frac{W_{pl}}{W_g}\right)\left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX}\right]\right\} \qquad \text{(T6)}$$

$$\frac{dW_g}{dX} = \frac{W_g}{C_{F,max}}\left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX}\right] \qquad \text{(T7)}$$

Constant payload- to gross-weight ratio:

$$\frac{1}{\mathcal{R}}\frac{d\mathcal{R}}{dX} = \frac{1}{I}\left\{\left(\frac{\partial I}{\partial X}\right)_{t_{ab},C_D} - \left[\frac{I}{C_F} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left(1 - \frac{C_F}{C_{F,max}}\right)\right]\frac{dC_D}{dX} - \left(\frac{\partial I}{\partial C_F}\right)_{X,C_D}\left[\left(\frac{\partial C_F}{\partial X}\right)_{t_{ab},C_D} - \frac{C_F}{C_{F,max}}\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D}\right]\right\} - \frac{1}{k}\left\{\frac{1}{W_g}\frac{dW_e}{dX} - \frac{1}{C_{F,max}}\left(\frac{W_e}{W_g}\right)\left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX}\right]\right\} \qquad \text{(T8)}$$

$$\frac{dW_g}{dX} = \frac{W_g}{C_{F,max}}\left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{t_{ab},C_D} - \frac{dC_D}{dX}\right] \qquad \text{(T7)}$$

TABLE II. - ACCELERATION-POTENTIAL EXPRESSIONS

[Evaluate at cruise combustor temperature except terms involving $C_{F,max}$.]

(a) Fixed-size airplane with constant initial gross weight

General equation:

$$\frac{dG}{dX} = \left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{C_D} - \frac{dC_D}{dX}\right]\left(\frac{1}{\frac{L}{D_t}}\right)\frac{1}{C_F} \qquad \text{(T9)}$$

(b) Fixed-size airplane with variable initial gross weight

General equation:

$$\frac{dG}{dX} = \left[\left(\frac{\partial C_{F,max}}{\partial X}\right)_{C_D} - \frac{dC_D}{dX} - \frac{C_{F,max}}{W_g}\frac{dW_e}{dX}\right]\left(\frac{1}{\frac{L}{D_t}}\right)\frac{1}{C_F} \qquad \text{(T10)}$$

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TABLE III. - ASSUMED SCHEDULE OF INLET-DIFFUSER

PRESSURE RECOVERIES AND AREA RATIOS

Flight Mach number, M	Pressure recovery, $\mathscr{P}$	Area ratio,[1] A_0/A_e		$\frac{A_0/A_e}{\mathscr{P}}$	
		Turbojet	Ram jet	Turbojet	Ram jet
1.0	0.95	0.711		0.748	
1.5	.944	.733	0.328	.787	0.347
2.0	.910	.822	.454	.903	.499
2.5	.826	.950	.644	1.15	.780
3.0	.700	1.08	.880	1.54	1.26
3.5	.567		1.15		2.03
4.0	.443		1.45		3.27
5.0	.245		2.00		8.16

[1]A_e is taken as compressor frontal area for the turbojet and combustor flow area for the ram jet.

TABLE IV. - NOZZLE PRESSURE RATIO AND AREA RATIOS

(a) Turbojet engine

M	t_{ab}, ^{o}R	$\frac{P_N}{p}$	$\frac{A_{th}}{A_e}$	$\frac{A_{exit}}{A_e}$ (a)
0.9	(b)	4.847	0.571	0.787
	2500	4.782	.635	.869
	3500	4.622	.797	1.084
1.0	(b)	5.293	0.566	0.818
	2500	5.223	.628	.901
	3500	5.042	.789	1.122
1.5	(b)	7.872	0.586	1.056
	2500	7.769	.652	1.165
	3000	7.620	.740	1.325
	3500	7.481	.822	1.456
2.0	(b)	12.13	0.569	1.331
	2500	11.97	.632	1.463
	3000	11.76	.716	1.671
	3500	11.57	.794	1.833
2.5	(b)	18.05	0.553	1.661
	2500	17.84	.611	1.821
	3000	17.51	.692	2.086
	3500	17.22	.769	2.289
3.0	(b)	25.43	0.535	2.014
	2500	25.15	.588	2.194
	3000	24.71	.664	2.514
	2500	24.30	.738	2.762

[a] For complete expansion.

[b] No afterburning.

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TABLE IV. - Concluded. NOZZLE PRESSURE RATIO AND AREA RATIOS

(b) Ram-jet engine

M	t_{ab}, °R	$\frac{P_N}{p}$	$\frac{A_{th}}{A_e}$ (a)	$\frac{A_{exit}}{A_e}$ (b)
1.5	1500	3.18	0.536	0.629
	2000	3.11	.649	.784
	2500	3.00	.770	.938
	3000	2.83	.915	1.109
2.0	1500	6.66	0.471	0.798
	2000	6.53	.564	.978
	2500	6.35	.658	1.150
	3000	6.15	.760	1.307
	3500	5.94	.879	1.459
	4000	5.70	1.0	1.634
2.5	1500	13.26	0.414	1.067
	2000	13.00	.492	1.300
	2500	12.79	.569	1.524
	3000	12.54	.647	1.733
	3500	12.29	.730	1.936
	4000	12.03	.824	2.141
3.0	1500	24.10	0.366	1.410
	2000	23.96	.433	1.711
	2500	23.76	.497	2.003
	3000	23.48	.560	2.280
	3500	23.15	.625	2.552
	4000	22.71	.694	2.826
3.5	2000	43.39	0.384	2.193
	2500	42.80	.439	2.569
	3000	42.17	.492	2.925
	3500	41.50	.546	3.276
	4000	40.55	.601	3.634
4.0	2000	66.05	0.344	2.725
	2500	65.71	.392	3.191
	3000	65.35	.439	3.639
	3500	64.91	.485	4.079
	4000	64.40	.531	4.524
5.0	3000	135.7	0.366	5.12
	3500	135.0	.402	5.76
	4000	134.3	.438	6.41
	4500	133.3	.465	7.05

[a]Values given for throat area are approximate.

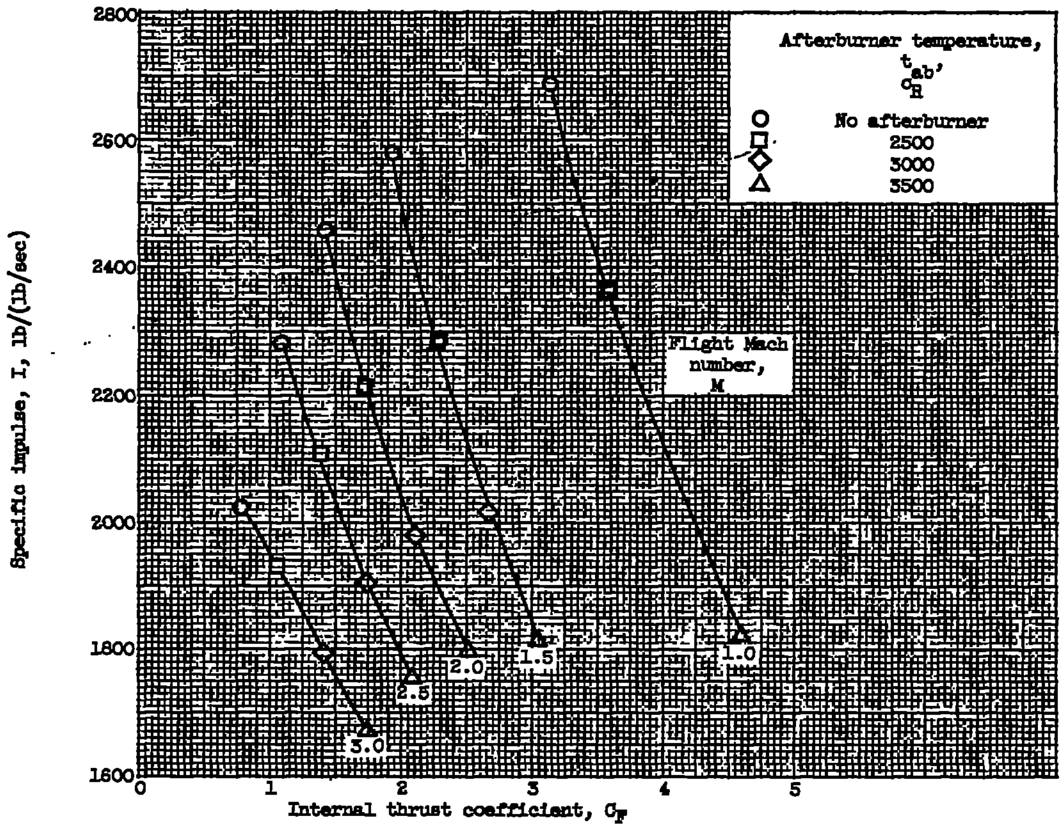
[b]For complete expansion.

TABLE V. - REPRESENTATIVE VALUES OF AIRPLANE PARAMETERS

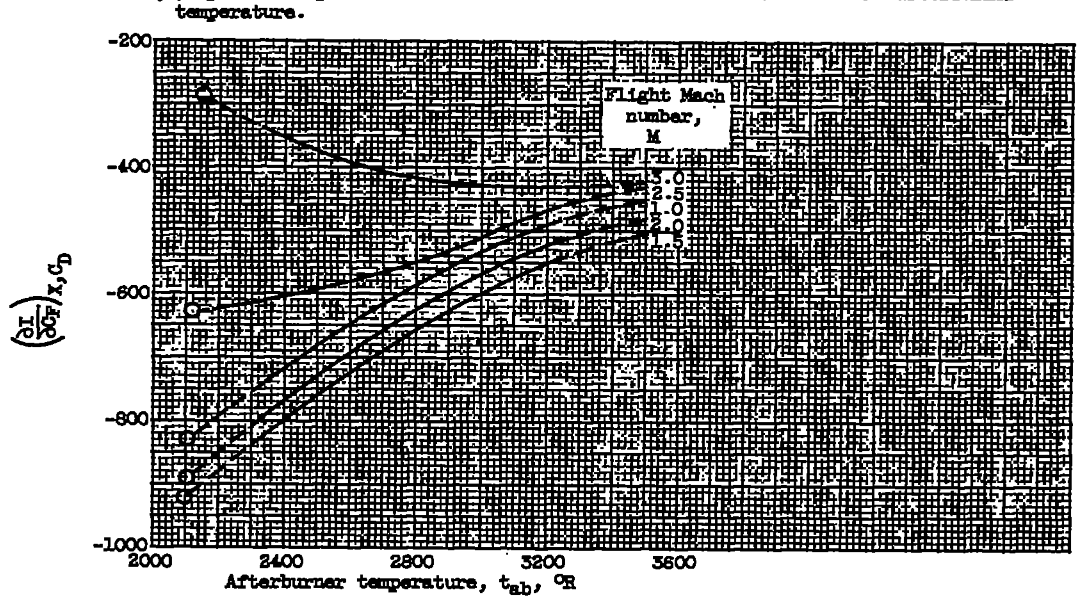
[Hydrocarbon fuel assumed.]

	Turbojet		Ram jet	
	M = 2.0		M = 3.0	
	Interceptor	Bomber	Interceptor missile	Bombardment missile
W_{pl}/W_g	0.10	0.07	0.25	0.07
W_{st}/W_g	.35	.22	.45	.21
W_e/W_g	.25	.16	.10	.06
W_f/W_g	.30	.55	.20	.66
$(W_f/W_g)_\alpha$	.12	.10	[a]0	[a]0
k	.16	.31	.18	.37
W_g, lb	20,000	150,000	10,000	100,000
L/D_t	4.0	5.5	3.0	5.0
Radius, nautical mi.	325	1500		
Range, nautical mi.			1000	5000

[a]The ram-jet missiles are assumed to be boosted by rockets or launched from another airplane at cruise speed and altitude.



(a) Specific impulse as function of internal thrust coefficient and afterburner temperature.



(b) Partial derivative of specific impulse with respect to internal thrust coefficient when change in afterburner temperature causes change in thrust coefficient.

Figure 1. - Turbojet-engine performance and performance derivatives.

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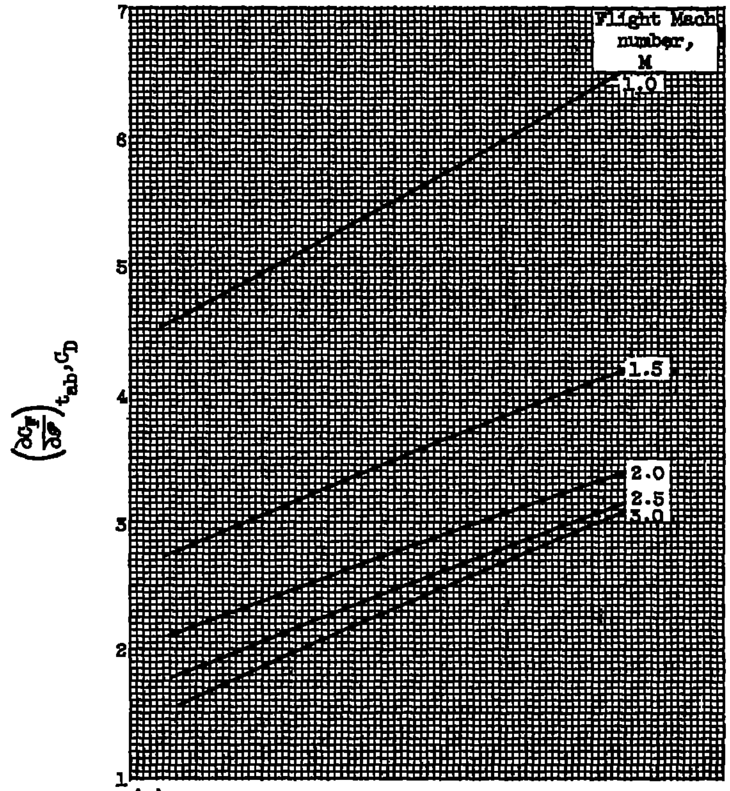
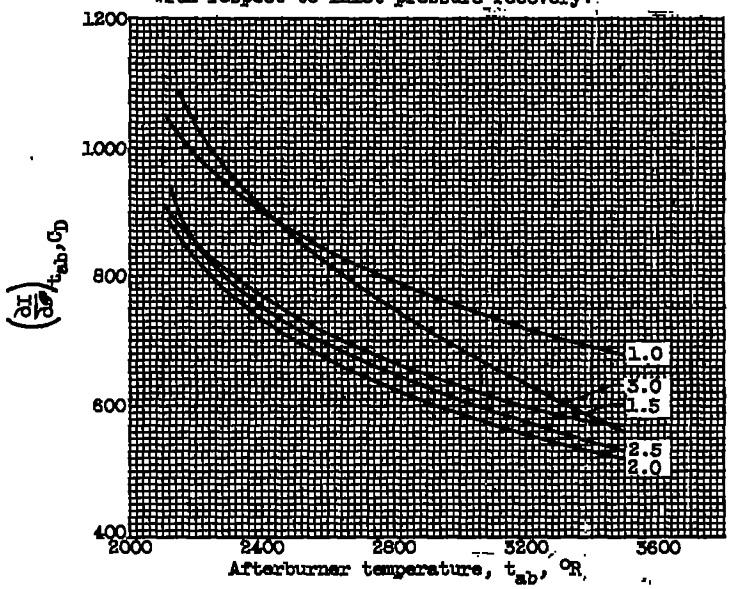
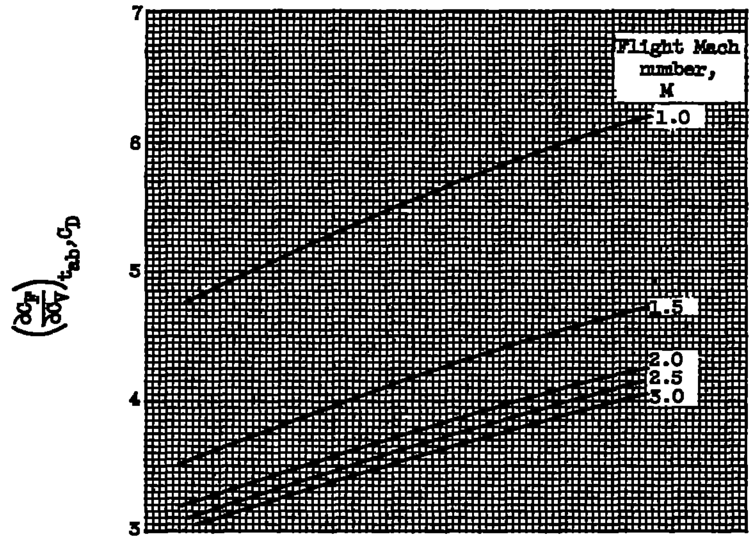


(c) Partial derivative of internal thrust coefficient with respect to inlet pressure recovery.

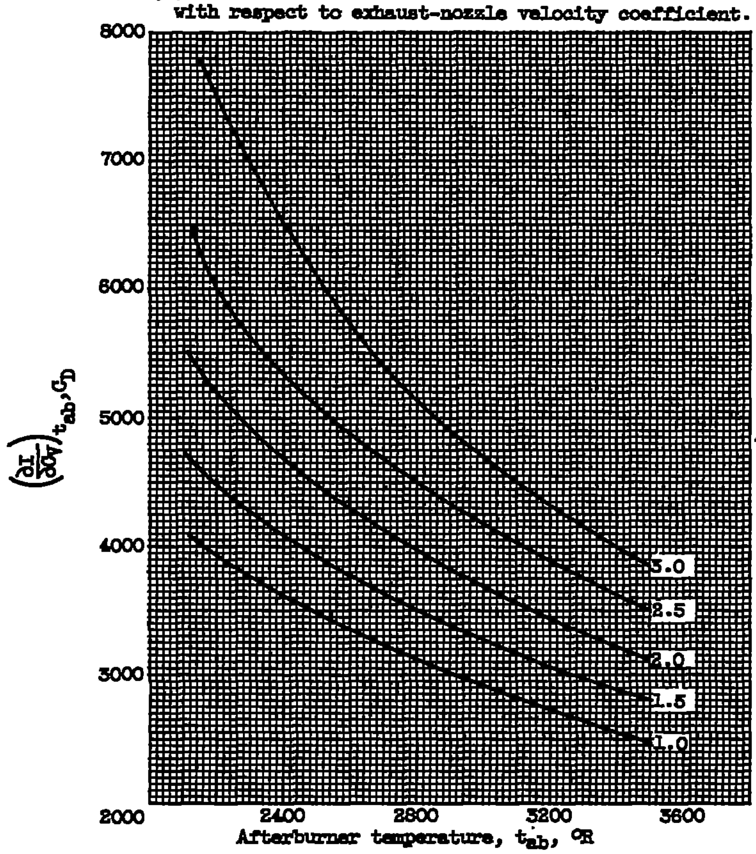


(d) Partial derivative of specific impulse with respect to inlet pressure recovery.

Figure 1. - Continued. Turbojet-engine performance and performance derivatives.

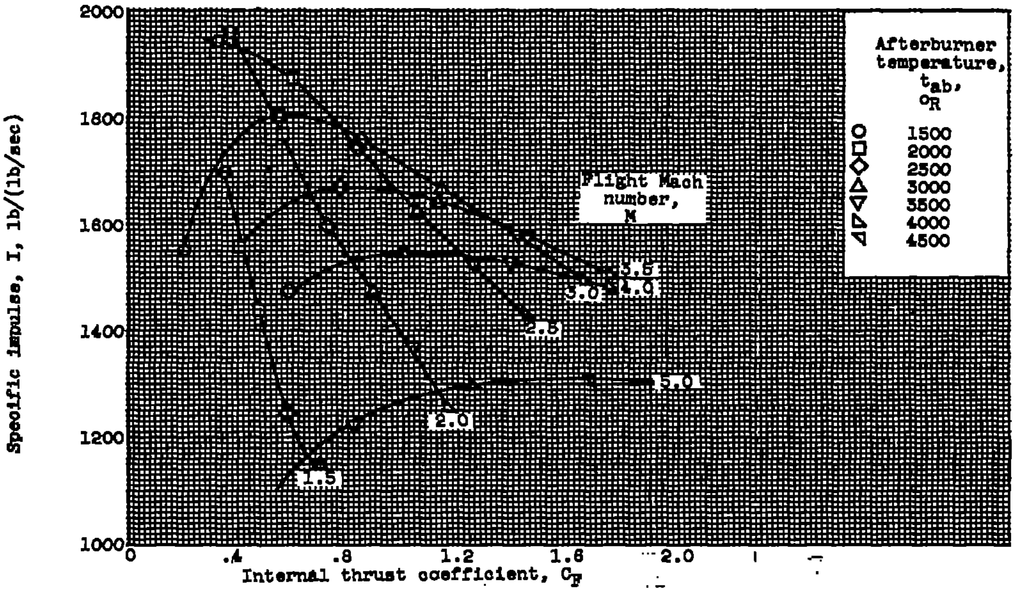


(e) Partial derivative of internal thrust coefficient with respect to exhaust-nozzle velocity coefficient.

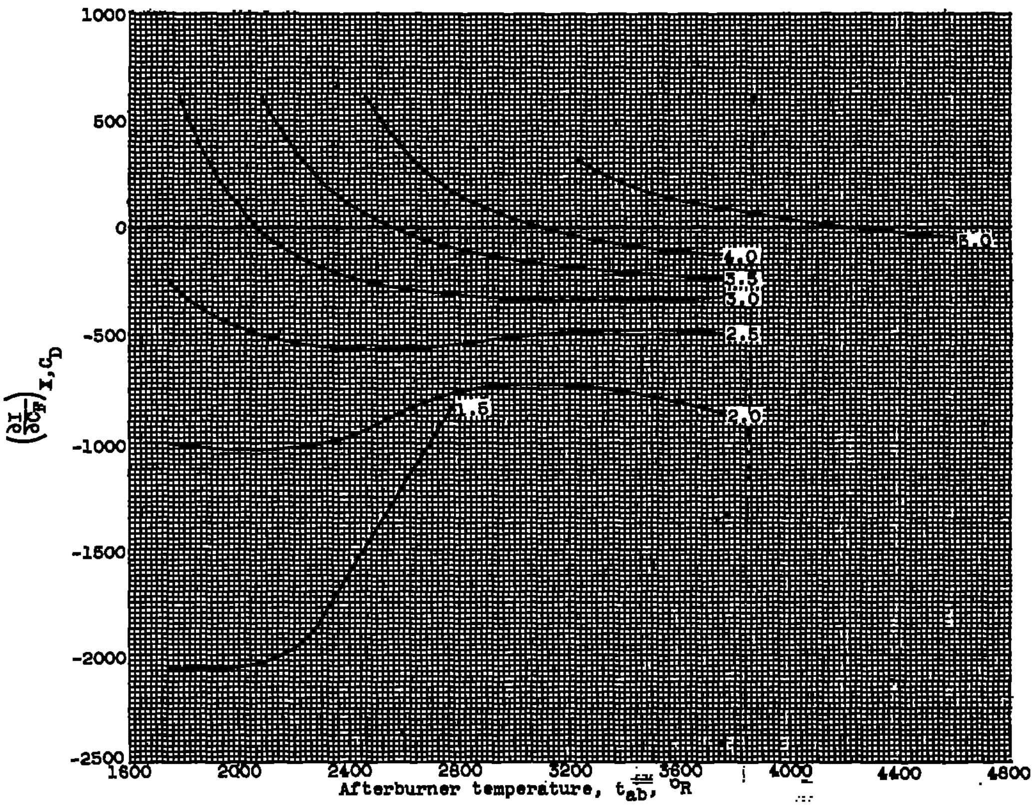


(f) Partial derivative of specific impulse with respect to exhaust-nozzle velocity coefficient.

Figure 1. - Concluded. Turbojet-engine performance and performance derivatives.



(a) Specific impulse as function of internal thrust coefficient and afterburner temperature.



(b) Partial derivative of specific impulse with respect to internal thrust coefficient when change in afterburner temperature causes change in thrust coefficient.

Figure 2. - Ram-jet-engine performance and performance derivatives.

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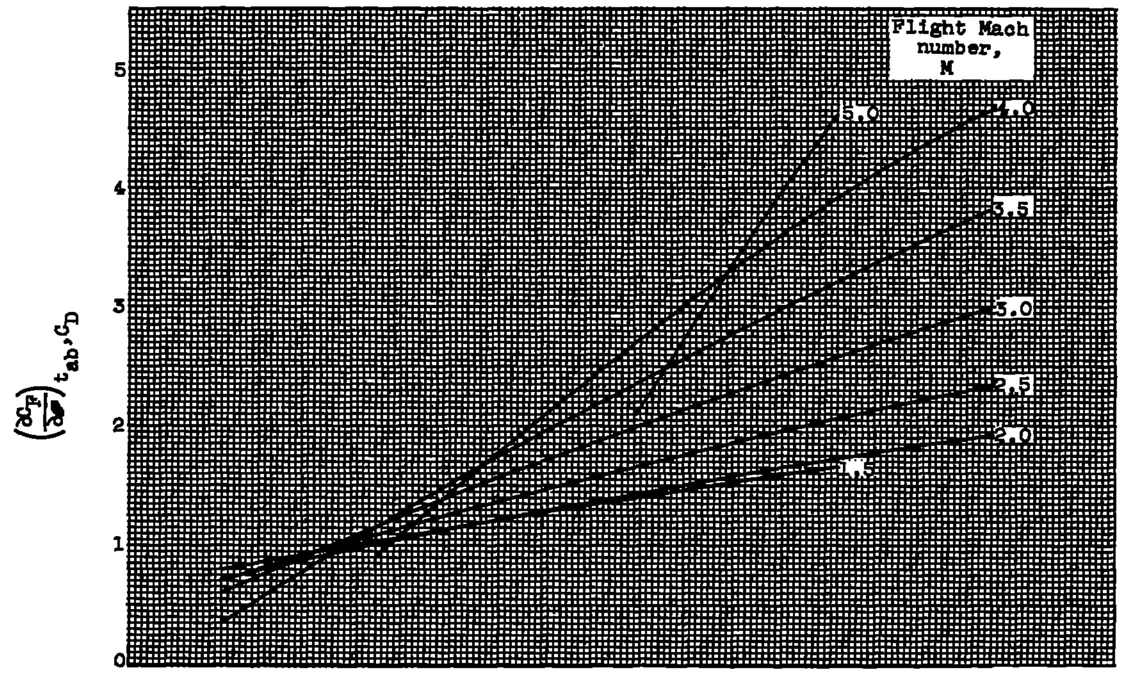
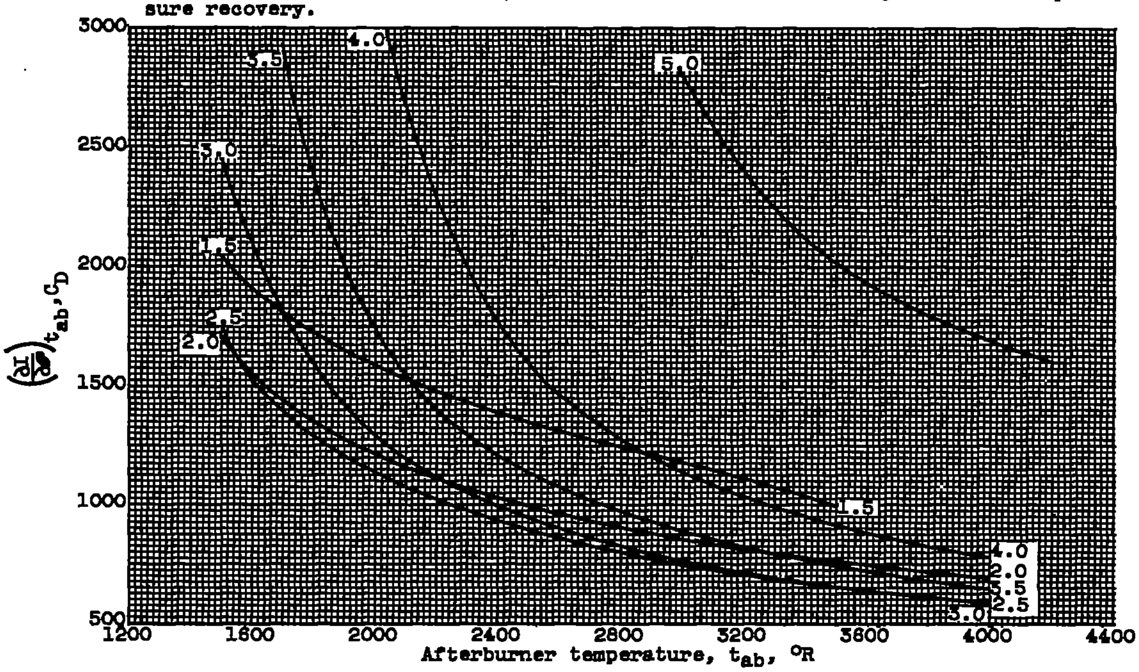


(c) Partial derivative of internal thrust coefficient with respect to inlet pressure recovery.



(d) Partial derivative of specific impulse with respect to inlet pressure recovery.

Figure 2. - Continued. Ram-jet-engine performance and performance derivatives.

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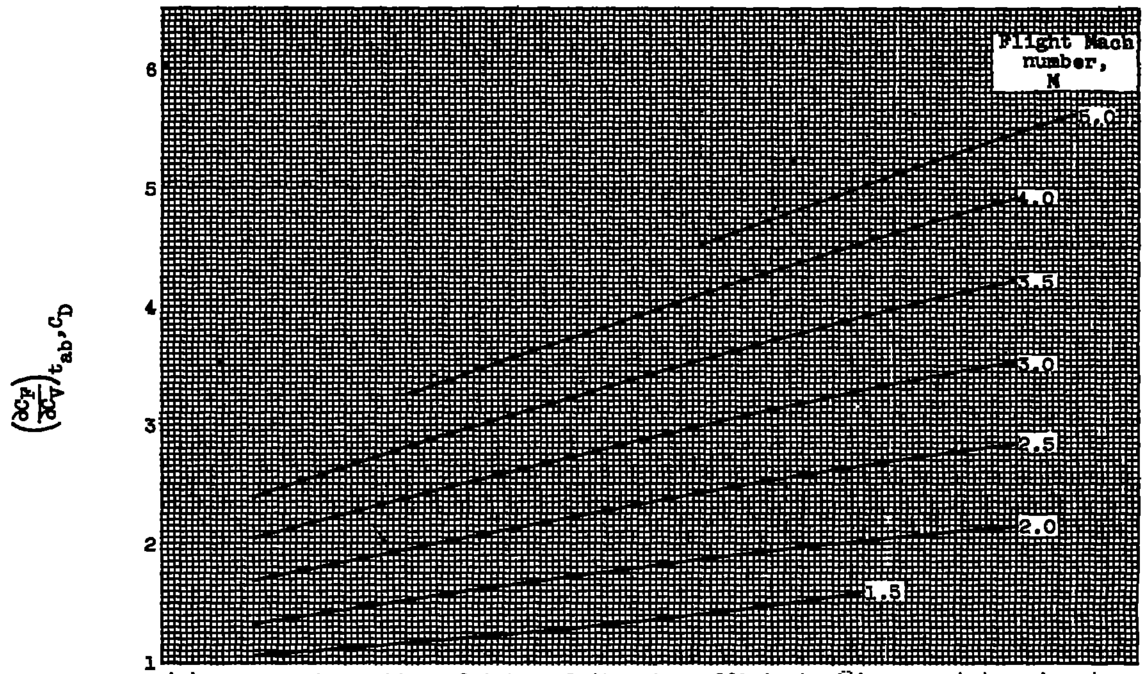
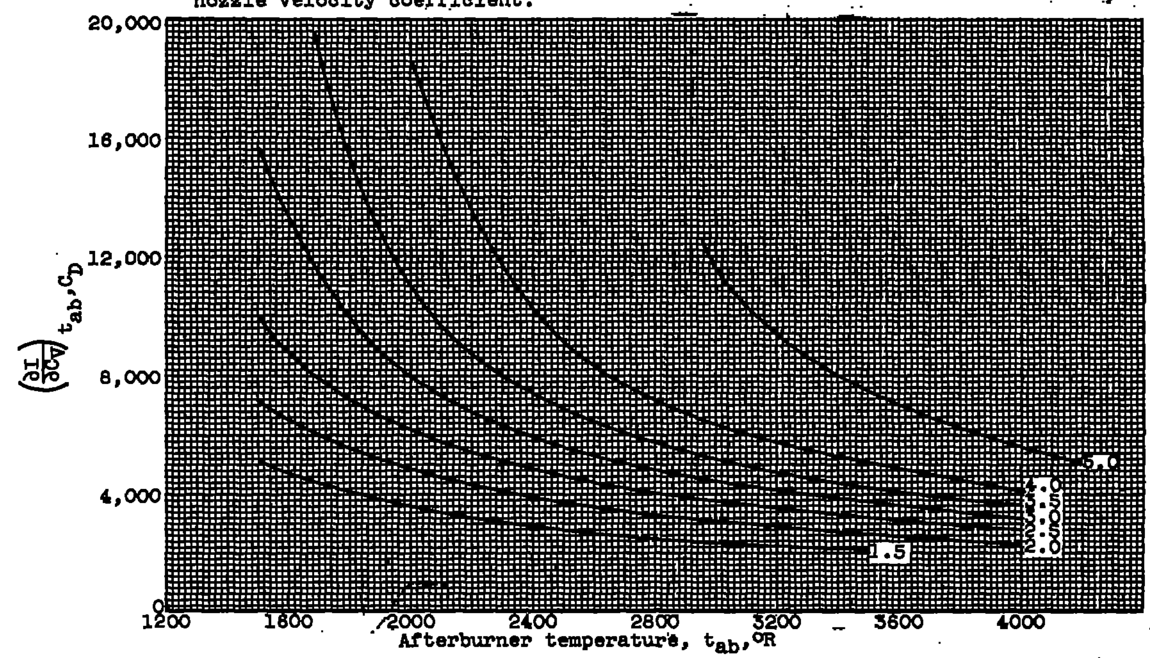


(e) Partial derivative of internal thrust coefficient with respect to exhaust-nozzle velocity coefficient.



(f) Partial derivative of specific impulse with respect to exhaust-nozzle velocity coefficient.

Figure 2. - Concluded. Ram-jet-engine performance and performance derivatives.

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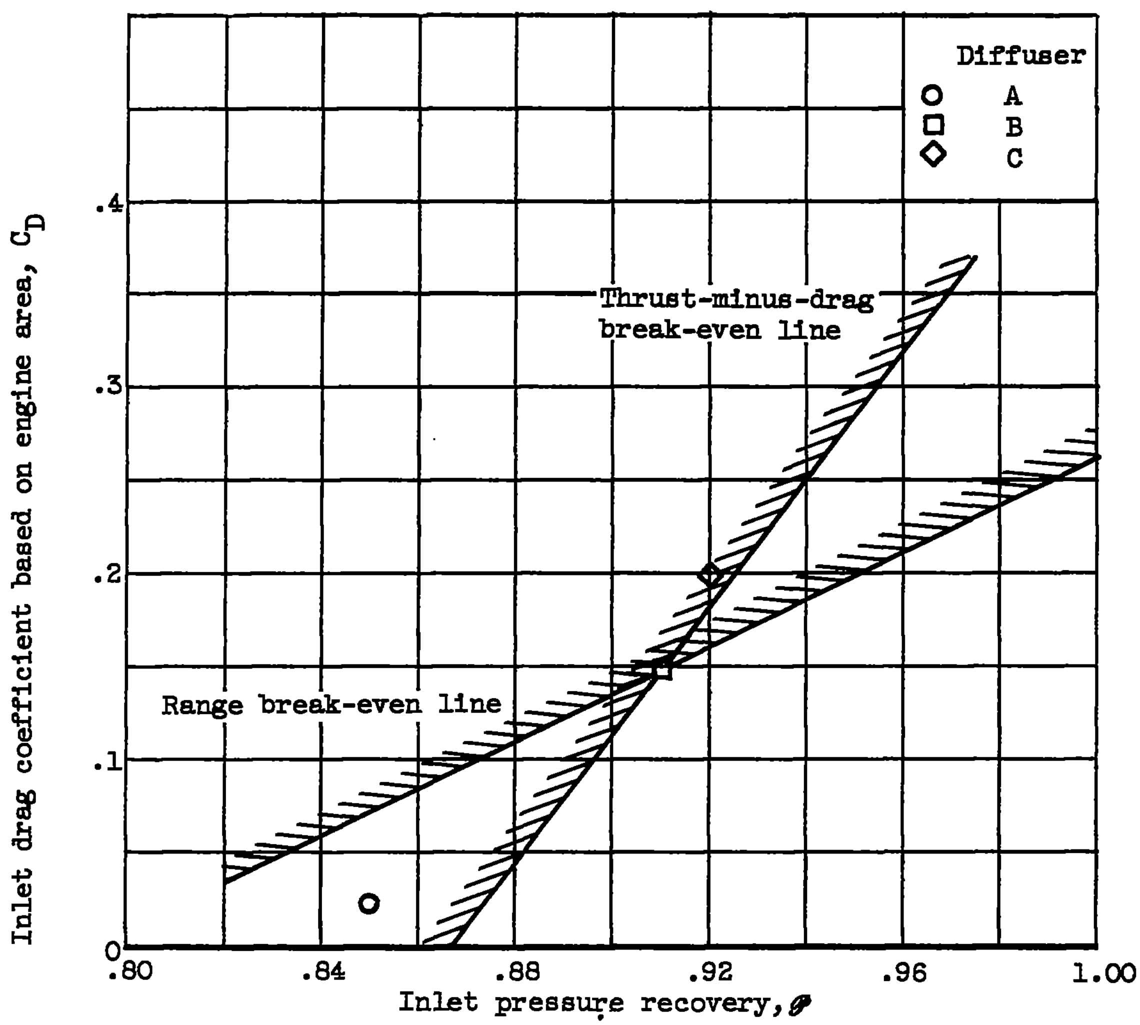


Figure 3. - Graphical method for comparing pressure recovery and drag characteristics of several inlets.

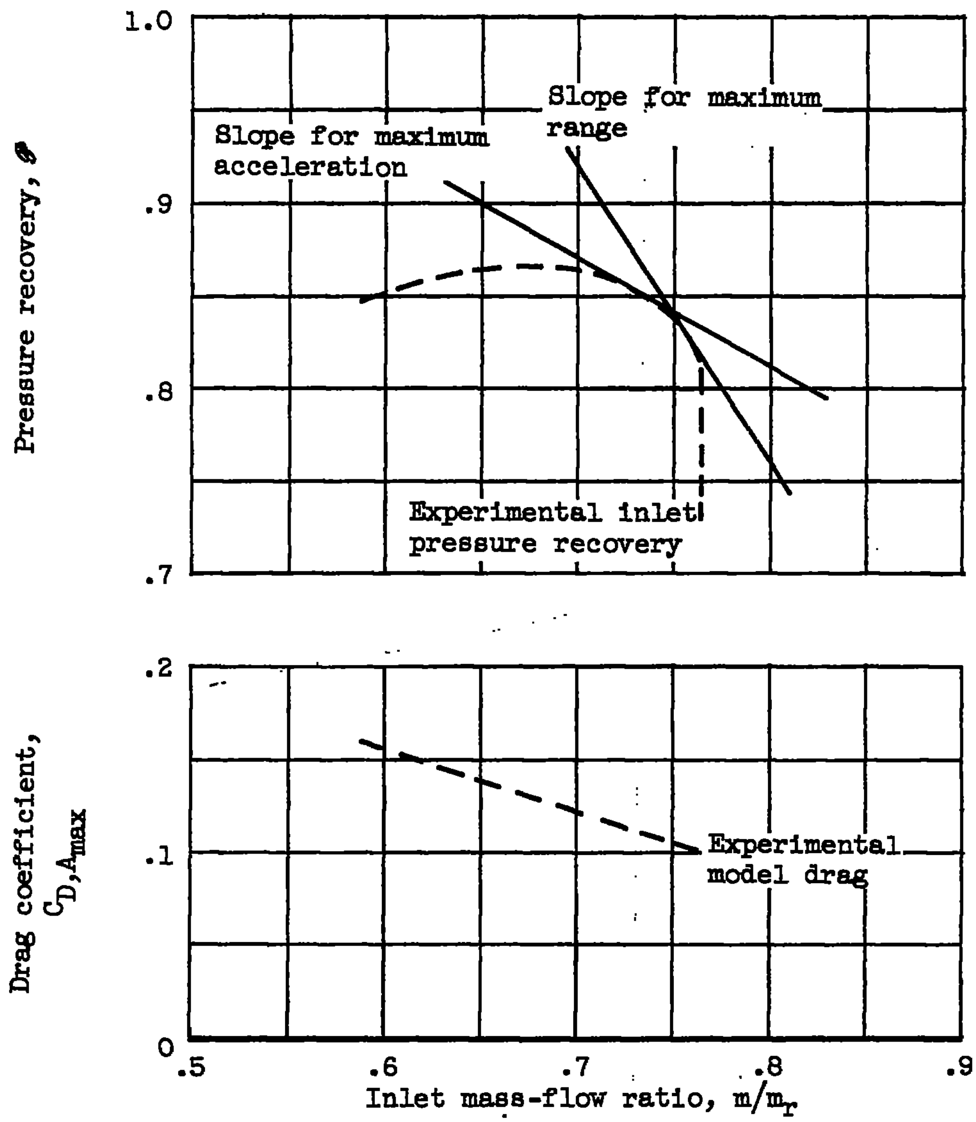
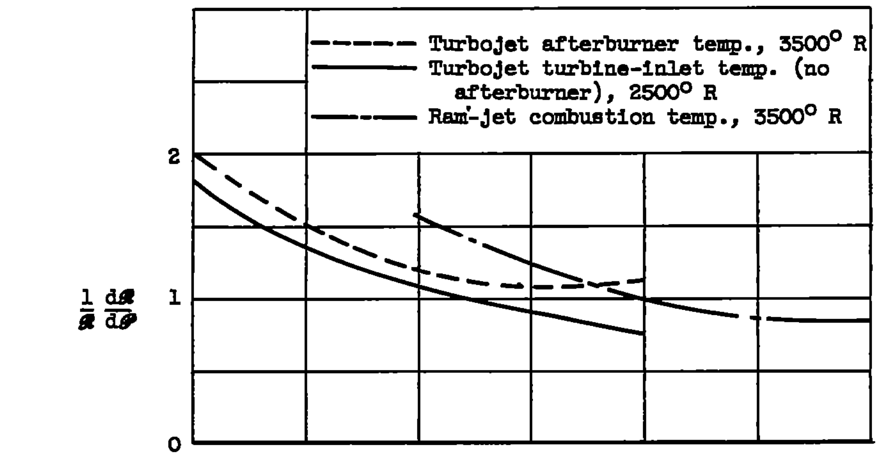
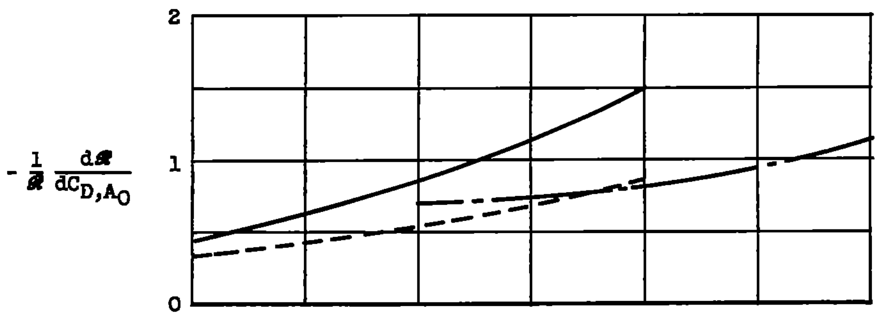


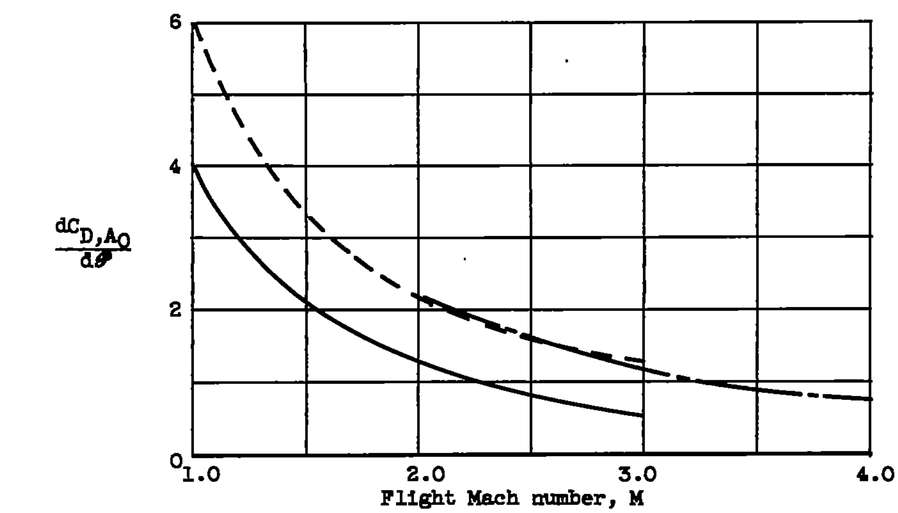
Figure 4. - Graphical method for selecting best inlet size and operating point. Flight Mach number, 2.0.



(a) Sensitivity of range to pressure recovery.

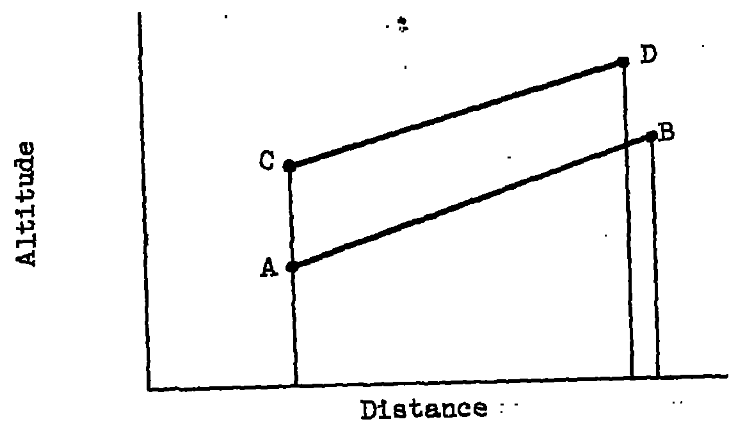


(b) Sensitivity of range to inlet drag coefficient.

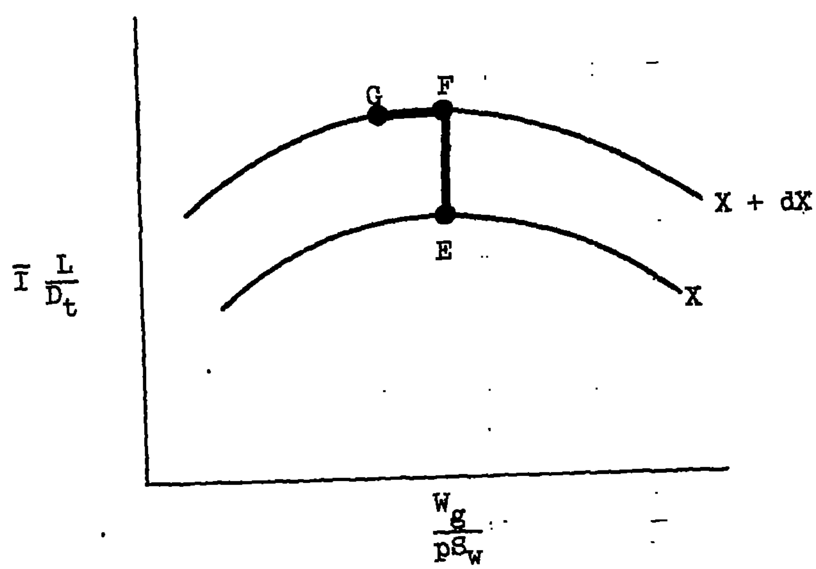


(c) Change in drag coefficient with pressure recovery for constant range.

Figure 5. - Relation of inlet pressure recovery and drag to range.



(a) Flight plans.



(b) Flight parameters.

Figure 6. - Flight characteristics.

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